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DIRECT RECORDING OF WIND SLIP

BY

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ABSTRACT

Methods of recording directly the relation of two wind speeds, S_1 and S_2, as a difference, $S_1 - S_2$, or as a ratio, S_1/S_2, are here outlined; the savings made possible by direct recording are summarized; and the limitations of the various equipments (at the present state of the art) are mentioned.

INTRODUCTION

In a wide variety of micrometeorological investigations, many of them concerned with the dissemination of aerosols, wind shear is an important parameter. Determination of wind shear is commonly performed by measuring the wind speed at two points, their spatial relationship being known, determining either the difference between the two speeds ("wind slip"), or the ratio of the two speeds ("R value"), and then substituting this derived figure in a formula.

As customarily performed, this determination is a slow process, requiring the reduction of two wind-speed records (usually on Esterline-Angus charts), performance of a preliminary computation using the reduced data, and substitution of the resultant figure (wind slip or R value) in a formula. Errors, not necessarily compensating, are introduced in the chart-reduction and data-transferring processes; and serious errors may pass undetected, particularly during gusty wind regimes, due to lack of synchronism of the two wind-speed recorders. Additionally, chart reduction and computation consume so much time that the condition of interest is usually long past before the salient figures become available.

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To reduce this computational "lead time" from minutes or hours to a fraction of a second, so that the recorded data are immediately available in the field in usable form, some sort of a medium speed computer is necessary. For most meteorological purposes, speed is adequate if the figure presentation lags the occurrence by as much as $\frac{1}{4}$ second (average human response lag). This is several orders of magnitude faster than a girl with a pencil, but very much slower than the millimicrosecond-responding radar computers now in use. As a happy result of this relatively low speed requirement, great equipment simplifications are possible. Additionally, as many of the anemometers now in use (such as the Beckman and Whitley Model 170-2) give a voltage analog output, simple analog computers can be coupled to their outputs directly, greatly reducing field equipment and field power requirements.

Experiments and tests with a number of mechanical and electrical computers disclose that several are immediately applicable to wind slip problems, and that several more hold considerable promise of future utility in this and similar applications. Summary descriptions of this work follow.

WIND-SLIP COMPUTATION

Preliminary Statement

The term "wind slip," as used here, is the difference between two wind speeds, measured in the same units, and is expressed mathematically as $S_1 - S_2$. Workable methods of computing wind slip automatically include one which is entirely mechanical, one which is electromechanical, and one which is entirely electrical. Limitations of each method are stated in the description.

Mechanical Method

Mechanical means for determining the difference between the speeds of two rotating shafts, such as anemometer shafts, consist of a planetary differential gear assembly, in which the two sun gears are driven in opposite directions by the respective anemometers. Rotation of the planet-gear carrier, in this arrangement, is a function of the speed difference. If an electric (or other) tachometer is driven by the carrier gear, voltage output of the tachometer will also be a function of the speed difference, and this can be indicated or recorded directly on a suitably calibrated instrument. General arrangement is shown schematically in Fig. 1.

Like many other primitive computing devices, this difference determiner works and works very well. For most field applications, however, it is somewhat unwieldy; and, where the anemometers are separated by any great distance, mechanical loading due to the interconnecting shafts seriously impairs the accuracy of the indicated wind speeds and speed differences.

All of these difficulties can be overcome, but not cheaply, by use of telemetering anemometers. Amplified signals from these drive servo motors, which in turn drive the planetary differential. This arrangement reduces anemometer loading to a negligible value, so that accuracy is not impaired by use of the difference determiner.

Similarly, the planetary differential, with both sun gears rotating in the same direction, can be used to indicate or record the sum of two wind speeds, or their average, as suggested informally by the late Dr. C. F. Brooks in 1942. As the number of planetary differentials needed to average 2^n wind speeds is $2^n - 1$, an extensive system of this type would be a mechanical or electromechanical nightmare; and a battery of capacitor-discharge recorders (**1**, **2**),[2] using a "common tank" developed at Dugway Proving Ground in 1945 (**3**, **4**), was found more practicable for field use.

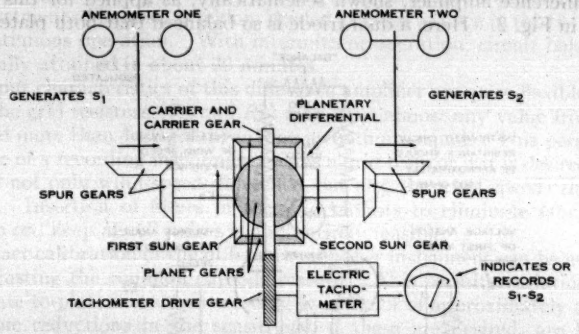

FIG. 1. Schematic diagram of mechanical difference determiner.

Electromechanical Method

When slow-rate contacting anemometers such as the Signal Corps type ML-80 are used, wind slip can be computed electromechanically, using a bidirectional stepping switch such as the Guardian type IR-R. A. S.

As employed, one anemometer steps the switch forward and the other steps it backward. In consequence, the position of the switch arm is an indicator of the cumulated difference between the two wind speeds. If this difference is read off at regular intervals, determined by the anemometer constants, wind slip in miles per hour is determinable.

Although theoretically simple, this method has serious practical limitations due to the schizophrenic characteristics of most bidirectional

[2] The boldface numbers in parentheses refer to the references appended to this paper.

stepping switches when supplied with simultaneous or near-simultaneous forward and backward signals. To eliminate this trouble, which is statistically important when the two wind speeds are similar (usual case), anticoincidence circuits and impulse harbors were developed. Although these could be made to work for thousands of operations without a misfunction, the amount of skilled maintenance required in the field was inordinate, so that the electromechanical wind-slip computer was shortly replaced by a variety of electrical computers, working directly from the voltage analogs of the wind speeds.

Electrical Method

With modern anemometers, whose output is a voltage analog of their rate of rotation, wind slip can be indicated or recorded directly by several simple and dependable electrical means. Most widely used of these is the difference amplifier, shown schematically, as applied for this purpose, in Fig. 2. Here, a dual triode is so balanced that both plates are

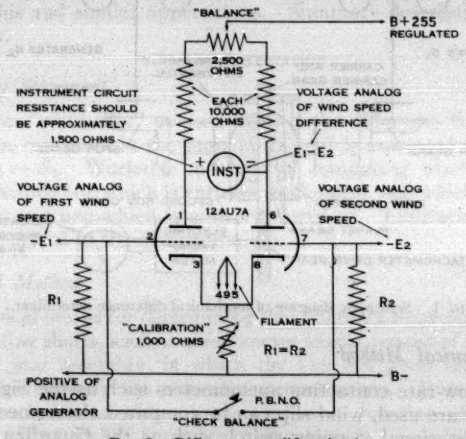

FIG. 2. Difference amplifier circuit.

at the same potential when both grids are equally biased. In consequence, an instrument connected from plate to plate will read zero. If, now, the bias on one grid is changed, the two plates will no longer be at the same potential, and an instrument from plate to plate will no longer read zero. So long as the bias voltages are within the limits of the straight line portion of the tube characteristic (here roughly between zero and − 10 volts), the instrument indication will be a function of the difference between the voltages applied to the two grids.

As meteorologically applied, bias voltages applied to the two grids

are analog voltages produced by two anemometers, and the plate-to-plate instrument can be either an indicating milliammeter, or a recording milliammeter, such as an Esterline-Angus 1-milliampere recorder.

Dependability and stability of this circuit are rather remarkable. Accuracies to within ± 1 per cent are easily attainable with either battery operation or simply regulated supplies, using a constant-voltage filament transformer for the filaments and conventional voltage regulator tubes for the plate supply. Additionally, percentage of error of instrument reading produced by a plate-voltage change is very much less than percentage of change in plate voltage. Life of standard "over the counter" tubes in this service exceeds 5,000 hours, and some "premium" tubes, such as RCA "Special Red" dual triodes, nominally rated at 10,000 hours, have a useful service life exceeding—and perhaps greatly exceeding—five years of continuous service.

Although checking of circuit balance once each shift seems prudent, readjustment of balance is seldom needed as often as once in thirty days of continuous operation. With intermittent operation, circuit balance is usually attained in about 20 minutes.

Input characteristics of this difference amplifier are quite flexible, so that the grid resistors ($R1$ and $R2$) may have almost any value from 1 ohm to more than 1 megohm, so long as both are equal. This permits the use of a recording instrument coil as a grid resistor if it is desired to record not only wind-speed difference, but also the wind speeds themselves. Insertion of filters in the grid circuits to eliminate 60-cycle pickup or "keep alive" pulses is also entirely feasible.

Exact calibration of the difference amplifier instrument can be made by adjusting the common cathode resistor. This permits variation of the plate-to-plate instrument reading by a factor of approximately two. Extreme reductions in the sensitivity, if these are desired, are best accomplished by use either of a shunt across the plate-to-plate meter, or of voltage-divider inputs.

Although specifically designed for determining the difference between two continuously generated voltage analogs, this difference amplifier can also indicate or record the difference between two simultaneous spot samples of wind speed (5), or between two integrated wind speeds (6), covering the same time interval.

Within reasonable limits, nonlinearities in the characteristics of the voltage analog generators on the anemometers can be partially corrected for by use of "Hallmark," "bootstrap," and other "sliding bias" circuits (some of them still classified) at the inputs of the difference amplifier.

SPEED-RATIO COMPUTATION

Preliminary Statement

Determination of the ratio of two wind speeds S_1/S_2, automatically, is a somewhat more difficult task for an electronic computer than is

determination of the difference of two speeds. As with the difference determination, several methods are available and can be made to work, but only a few are suited to current meteorological needs. Only two of these methods will be outlined here—the bridge divider, a direct process, and the logarithmic subtractor, an indirect process.

Bridge Divider

In any bridge circuit, when balance is attained so that the instrument reads zero, the ratio of resistances is:

$$\frac{R_1}{R_2}=\frac{R_3}{R_4}. \tag{1}$$

If, with all other resistance values variable, we fix R_4 at any value K, relation (1) becomes:

$$\frac{R_1}{R_2}\sim R_3 \tag{2}$$

and we have an electrical means of determining ratios. This is true whether the bridge is energized with a.c. or with d.c., and regardless of the magnitudes of the resistances concerned.

Conversion of a simple bridge into an automatic ratio computer with continuously recording output is theoretically quite simple, electromechanically feasible at the present state of the art, and not impossibly expensive.

To make this conversion, the two input resistors, R_1 and R_2, are replaced by servo-adjusted variable resistors, so arranged that their values are direct functions of the applied wind-speed analog voltages. The instrument (Fig. 3) is replaced by a null-seeking servomechanism, which adjusts R_3 to balance the bridge whenever an unbalance exceeding a given (very small) unbalance occurs. To the servo shaft is coupled a recorder, calibrated in terms of R_3/R_4, which is also the desired ratio R_1/R_2. As an automatic self-adjusting bridge requires a finite (but very short) time to rebalance whenever a change in conditions occurs, a "print when balanced" pen control is highly desirable, to produce a clear record, uncluttered with "seeking rebalance" traces.

At the present state of the art (Dec. 1959) such a ratio recorder can be constructed in about 90 days, at a cost approximating $3500, by any one of half a dozen reputable manufacturers of servo-controlled bridges. Weight will approximate 75 pounds, power consumption (at 115 volts, 60 cycles) will approximate 100 watts, and operating time between maintenance shutdowns will probably not exceed 5000 hours divided by the sum of the number of tubes and vibrators contained in the assembly. With conventional designs, this indicates the need for a maintenance

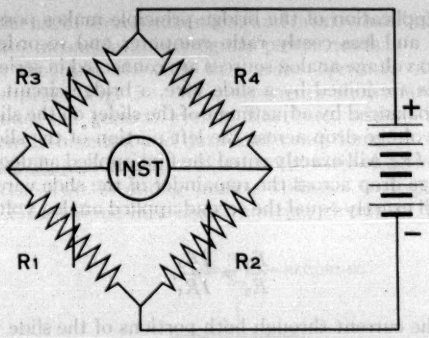

FIG. 3. Resistance relations in a simple bridge.

shutdown roughly every ten days. Average operating error in a device of this type can be held to somewhat less than 1 per cent under "industrial" operating conditions, and to better than 0.1 per cent under rigorously controlled laboratory conditions, where a highly skilled (and costly) operator and maintenance man are continuously on duty.

Simplification of this arrangement, leading to the elimination of the two servo-controlled input resistors, is not currently feasible, as no standard vacuum tube or transistor has a through resistance which is a linear function of the applied bias only. A slightly happier situation prevails if we use an a-c. bridge, substituting reactance for resistance. Nearly linear saturable reactors are commercially available, and the saturation current, which varies the reactance, can be the plate current of a power pentode (such as a 6L6 or 5881), which, through a reasonable range of applied grid voltages, is nearly a direct function of grid bias. Final accuracy of such a bridge is limited by phase-shift errors, which are at a minimum when the two inputs (wind-speed voltage analogs) are equal.

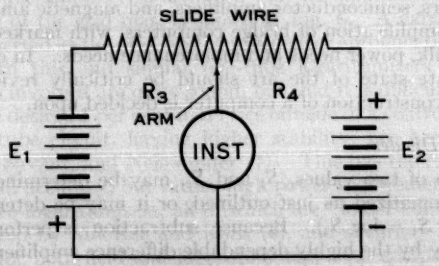

FIG. 4. Slide wire bridge circuit.

Another application of the bridge principle makes possible a somewhat simpler and less costly ratio computer and recorder. If, as in Fig. 4, the two voltage analog sources are connected in series aiding, and their free ends are joined by a slide wire, a bridge circuit is produced, which can be balanced by adjustment of the slider on the slide wire. At balance, the voltage drop across the left portion of the slide wire, here designated as IR_3, will exactly equal the first applied analog voltage, E_1; and the voltage drop across the remainder of the slide wire, here designated IR_4, will exactly equal the second applied analog voltage, E_2. In consequence,

$$\frac{E_1}{E_2} = \frac{IR_3}{IR_4}. \tag{3}$$

Now, as the current through both portions of the slide wire is equal when the bridge is balanced, relation (3) may be simplified to

$$\frac{E_1}{E_2} = \frac{R_3}{R_4}. \tag{4}$$

With this relationship, the ratio may be computed and recorded automatically by use of a single self-balancing bridge, the arm position being calibrated in terms of R_3/R_4. A relatively simple self-balancing bridge, such as the highly dependable, but now obsolete, Leeds and Northrup Micromax Recorder, is entirely adequate for this purpose.

Cost of such a bridge, commercially made, having an average error of less than 0.5 per cent, a weight of less than 50 pounds, a power need (at 115 volts, 60 cycles) of about 40 watts, and an interval between breakdowns of at least 1000 hours, is about $1250, with a strong chance of an even lower price if a large number of similar devices are built at the same time. Ratio range coverable by an instrument of this type is from 0.1 to 10.0, which is slightly more than is usually needed in micrometeorology.

Upcoming electronic advances, especially in the fields of photocell galvanometers, semiconductor amplifiers, and magnetic amplifiers, hold promise of simplification of bridge computers, with marked reductions in weight, bulk, power needs and maintenance needs. In consequence, the immediate state of the art should be critically reviewed before purchase or construction of a computer is decided upon.

Logarithmic Divider

The ratio of two values, S_1 and S_2, may be determined by direct division, automatized as just outlined, or it may be determined from Antilog $(\log S_1 - \log S_2)$. Because subtraction is performable with great rapidity by the highly dependable difference amplifier, previously outlined, this method is quite attractive, particularly as its employment

eliminates the use of servomechanisms and print-when-balanced controls, with their not inconsiderable irregular power needs.

A block diagram, showing the *modus operandi* of a logarithmic divider, comprises Fig. 5. All components shown are commercially available, and the general system has been widely tested under a variety of environmental and data-load conditions.

First step in performing a logarithmic ratio determination is conversion of the two incoming analog voltages to logarithmic analog voltages. This is done in a logarithmizing circuit, of which there are

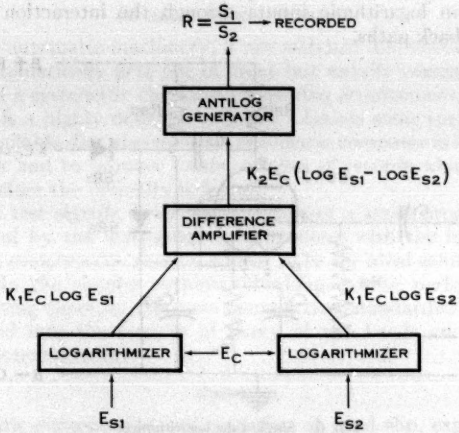

FIG. 5. Block diagram of logarithmic divider.

several workable varieties. A simple active circuit, which is simply a vacuum tube amplifier so arranged that its amplification varies as the logarithm of the grid bias, is shown in Fig. 6. A carrier signal (a-c.) is here inserted at the grid, and rectified after amplification, at the plate. Tests with this and similar one-tube logarithmizers show that the principles involved are quite sound, but that it is difficult to keep errors below the desired 1 per cent in service outside of a controlled laboratory. A multi-tube circuit, having higher stability, has been published by Howard, Savant, and Neiswander (7). This has the requisite dependability and accuracy, but rather high power requirements.

Passive logarithmizers, all of which are "losser" devices, have been summarized by MacRae and Roth (8), and a variety of such devices, suitable for both field and laboratory use, with compatible amplifiers, is now commercially manufactured.[3] Because of relatively low power

requirements (despite the need for thermal control), high dependability, and long service life, passive logarithmizers are the best type currently available for this application.

Subtraction of the two logarithmized voltages is performed by a difference amplifier, already detailed.

Conversion of the resultant log ratio to ratio is theoretically quite difficult, as currently available antilog generators are quite complex, highly demanding of power, and require almost continual skilled maintenance and adjustment. Most devices of this type somewhat resemble a family of interconnected volume expanders (9), and give numerical outputs from logarithmic inputs through the interaction of several special feedback paths.

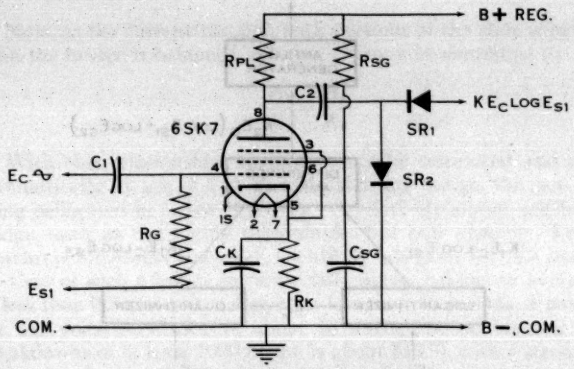

FIG. 6. Single-tube logarithmizer.

For most micrometeorological uses, where wind-speed ratio is commonly between 0.25 and 4, the output meter, connected from plate to plate of the difference amplifier, can be calibrated directly in ratios, and the troublesome and costly antilog generator dispensed with entirely. A zero-center Esterline-Angus instrument is ideal for this purpose, and special ratio paper, containing the requisite numerical conversion, can be obtained relatively inexpensively.

Possible Ditigal Systems

Demonstrably feasible, but not currently available "over the counter," are a number of digital and digital-mechanical devices, which are capable of converting a voltage input, such as an analog voltage, into a series of electrical signals, contact closures, or shaft rotations which can then be used to operate a conventional electromechanical

[3] Made by C. E. S. Electronic Products, Inc., P. O. Box 7504, San Diego 7, Calif.

computer. This can be designed to give an output as printed figures on a paper tape, coded punchings on a tape, or as a sequence of punched cards, immediately suitable for further mathematical processing.

Currently, such digital systems are too costly, too bulky, too demanding of power, and too often out of order to be usable at any but the largest installations, where skilled maintenance is immediately available. Development work now in progress indicates that a marked improvement in this situation will probably take place within the next few years, so that a reconsideration of digital systems will be in order about 1965.

AUTOMATIC SYSTEM CHECKING

Because automatic machinery, if not watched and checked systematically, automatically gets out of order but usually continues to run, some sort of a systematic checking and testing arrangement, preferably automatic, is a highly desirable adjunct. Unless some such system is not only available, but also used, an automatic computer is likely to get out of order and to produce yards or miles of records which are pure gibberish before the difficulty is found.

Periodic test signals, inserted by means of a small program timer, and recorded by the instrument system along with the instrumental records and computation results, appear to be the ideal solution for this problem. In the simpler systems, checking can be performed by a small repeating timer, in the same manner that standardization signals are switched into the circuits of many of the Leeds and Northrup recording potentiometers.

SUMMARY

Automatic computation and recording of wind slip, expressed as a speed difference, are currently performable by means of a recording difference amplifier, a simple, stable, dependable, and inexpensive electronic device suitable for continuous service in both the laboratory and the field.

Automatic computation and recording of wind-slip ratio are best performed at present by the logarithmic method, using passive logarithmizers, followed by a difference amplifier, which feeds a specially calibrated recorder. This arrangement is marginally operable from batteries, and is suitable for continuous commercial operation from any ordinary a-c. supply line.

Entirely feasible, but currently "priced off the market," is a variety of bridge computers which determine wind-slip ratios automatically and directly. Utility of these devices may be increased in the near future by improvements now under development.

Several digital and digital-mechanical computers can be made to perform wind-slip computations with requisite accuracy and speed, but

at relatively high costs. Reconsideration of digital computers for this work seems in order after completion of improvements and simplifications now in process.

Automatic system checking, with the check signals forming a part of the instrument record, is strongly recommended for any and all automatic meteorological computers.

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THE NODE SYSTEM OF EQUATIONS

BY

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ABSTRACT

The minimum number of simultaneous equations and unknowns required to describe the behavior of an electrical network is in many cases obtained by considering the independent node voltages to be the unknown variables. However, the usual techniques for writing node equations are not applicable to networks containing ideal voltage sources not incident to the reference node.

The node system of equations for general networks with ideal current and voltage sources arbitrarily located, is derived on the basis of a particular type of cut-set or segregate matrix, and a novel method for writing the node equations of such networks by inspection is presented. The application to networks containing vacuum tubes, transistors and magnetic coupling is illustrated.

The behavior of an electrical network may be completely described in terms of its driving functions, its parameters, and a linearly independent set of voltage or current variables. If one junction point or node in the network is chosen as a reference node, then it will be shown that some set of voltages between other nodes and this reference constitutes such an independent set; the corresponding set of equations for a linear network is here referred to as the node system of equations. An alternate independent set of variables is called the mesh or loop set of currents, which appear in the mesh system of equations. The number of linearly independent equations in each of these two systems is not generally the same. The node system involves the smaller number of equations and unknowns in the case of certain networks, and other advantages for the node system of equations have also been cited (1, 2).[3]

The usual techniques for writing node equations are not directly applicable to networks containing ideal voltage sources not directly connected to the reference node. Equation sets have been written involving an added current variable for each such ideal voltage source (3, 4), however, these sets of equations should be called hybrid systems since they contain both voltage and current variables.

This paper presents a derivation of the node system of equations in which the unknown variables are exclusively an independent set of node voltages. A new type of matrix called a node segregate matrix is introduced. It is shown that this matrix together with a description of the elements making up the network provides sufficient information

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[3] The boldface numbers in parentheses refer to the references appended to this paper.

to permit formulating the node system of equations. Two methods of formulating the node system of equations are described and several examples are presented to illustrate the application of these methods to different types of networks.

Throughout the paper, currents and voltages are considered to be functions of time and the self and mutual admittances to be functions of the derivative operator p; however for brevity the functional notation is omitted.

NETWORKS AND ELEMENTS

A network is considered to be composed of a set of interconnected elements. The type of network diagram used here is a representation of the network in which elements appear as oriented line segments, which meet at junctions or nodes. The elements are considered to be exclusively ideal voltage and current sources, and self admittances. In many cases mutual admittances couple certain self-admittance elements. These mutual admittances are indicated by writing the appropriate symbol, such as Y_{jk}, between the graphical symbols for the self-admittance elements j and k which are coupled.

A transformer is represented by a number of self-admittance elements equal to the number of coils, and it is understood that two equal bilinear mutual admittances are present for every pair of magnetically coupled coils. A vacuum tube in its small signal or incremental mode of operation is represented as two self-admittance elements with a common node, coupled by at least one non-zero unilinear mutual admittance. Similarly, a transistor in its small signal mode of operation is represented as two self-admittance elements with a common node, coupled by two unequal unilinear mutual admittances. In effect, the transformer, tube and transistor are treated as "black boxes" with terminal relations described completely by self and mutual admittances. The use of internal generators in tube or transistor representation is avoided since these are superfluous when the admittances are known. The details of the methods for expressing admittance relations for transformers and vacuum tubes are available in the literature (5). An indefinite or redundant admittance matrix (6) is useful for specifying the admittance relations for different types of transistor arrangements, and this mathematical device can also be used with electronic tube circuits.

The general types of voltage and current sources usually considered in the literature are treated here in terms of the more basic elements which are the ideal dissipationless sources in series or parallel combination with passive elements. The method is found to permit a direct analysis of the given network with no added complication or increase in number of equations or unknowns and without the use of preliminary "exchange of sources" (7).

Each ideal voltage source connected between a pair of nodes is found to reduce by one the number of equations which would otherwise need to be solved simultaneously.

Precise identification of variables requires some method of polarity marking or orientation. An arrow marking on each element indicates directly the positive sense of element current, and the element voltage is oriented positively at the tail of the arrow. Any unoriented line segment in the diagram simply represents a short circuit.

BASIC CONCEPTS

The concept of the segregate (8), which is similar to that of the cut-set (9), is fundamental to the present study. The segregate current equations for a network may be considered a generalization of the Kirchhoff current law relations for the network. Node current equations are a specialized form of segregate current equations.

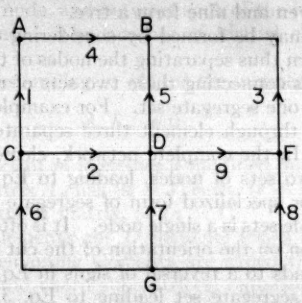

FIG. 1. A network diagram with elements represented as oriented line segments.

The network of Fig. 1 may be used to illustrate these concepts. In this diagram, the elements are numbered, but specification of element type has been intentionally omitted.

Summing the currents out of each of the nodes B and F yields the node current equations

$$-i_2 - i_4 - i_5 = 0 \tag{1}$$

and

$$i_3 - i_8 - i_9 = 0. \tag{2}$$

Elements four, five, eight and nine may be considered to form a segregate set (or cut-set), and the corresponding segregate current equation is

$$-i_4 - i_5 - i_8 - i_9 = 0. \tag{3}$$

It should be noted that segregate current equation (3) may be obtained by adding node current equations (1) and (2).

Evidently other node current equations can be written for the given network. It has been shown that $(n-1)$ independent node current equations can be written for any network (10), thus since $(e) > (n-1)$, the coefficients of the linearly independent node current equations may be arranged to form an $(n-1) \times (e)$ node current matrix of rank $(n-1)$. Since segregate current equations may be formed by elementary operations on the set of node current equations, the coefficients of the segregate current equations may be arranged in the form of an $(n-1) \times (e)$ segregate current matrix which also has the rank of $(n-1)$ (11). Thus for Fig. 1, the segregate current equations result in a five row by nine column segregate current matrix of rank five.

The concept of the network tree provides a systematic method for writing a useful set of segregate current equations. In general, any set of elements which involves all of the nodes but which does not contain a closed path, constitutes a tree. For Fig. 1 for example, elements one, two, three, seven and nine form a tree.

Segregate sets may be formed by considering each element of the tree to be cut in turn thus separating the nodes of the network into two sets. The elements connecting these two sets of nodes in the original network constitute one segregate set. For example, for Fig. 1 and the chosen tree, a cut through element three separates node B from the remaining nodes. In the complete network, elements three, four and five connect the two sets of nodes, leading to Eq. 1 which is a node current equation, or specialized form of segregate current equation in which one of the node sets is a single node. It is often useful to base the segregate orientation on the orientation of the cut element in the tree. Such orientation leads to a reversal of signs in Eq. 1. In the present example, a second segregate set leading to Eq. 3 results from a cut through element nine of the tree. Signs are again reversed if the orientation of element nine determines segregate orientation.

The node voltages of a network are defined as the voltages between each of $(n-1)$ nodes and the remaining or reference node, oriented toward the reference. There may however be less than $(n-1)$ independent node voltages. In particular only one node voltage associated with any subtree of ideal voltage sources in the network can be independent. As a preliminary to the development of the node equations, it is useful to augment the network by adding what will be called virtual elements located and oriented such that the voltages across these virtual elements correspond to the independent node voltages. These virtual elements are treated as zero admittance passive elements and designated by the symbol O with identifying subscripts. These are not counted in the number (e) of original elements.

DERIVATION OF THE NODE SYSTEM

The Kirchhoff current law and voltage law relations for a network may be respectively expressed in the form of matrix equations as

$$AI_e = 0 \tag{4}$$

and

$$BV_e = 0. \tag{5}$$

These equations may be called the node current matrix equation and loop voltage matrix equation, respectively. A is a node current matrix of $(n-1)$ rows, (e) columns, and rank $(n-1)$. B is a loop voltage matrix of $(e-n+1)$ rows, (e) columns, and rank $(e-n+1)$. I_e is a column matrix of the element currents, (e) in number, and V_e is a column matrix of element voltages, also (e) in number.

It is useful to define a tree of the network, and to specify that all ideal voltage source elements are included in the tree. There may then be a number of I_E element subtrees within the complete tree and within the network. The network is augmented by adding one type O or virtual element between one arbitrarily selected node in each such subtree and the reference node, and between each of the remaining $(n-1)$ nodes and the reference. The number of virtual elements added is evidently $n_O = n - 1 - n_e$.

The equation corresponding to (4) for the augmented network is

$$A_m I_{em} = 0 \tag{6}$$

where the added subscript m identifies a modified matrix in each case. The modified node matrix A_m has $(n-1)$ rows, $(e+n_O)$ columns and a rank the same as for the A matrix, or $(n-1)$.

The union of I_E elements and O elements forms the complete network tree, and the remaining elements are called the chords. A_m is partitioned into tree and chord submatrices, requiring a corresponding choice of element order in I_{em}, and column order in A_m. Therefore Eq. 6 may be written as

$$\|A_T \vdots A_C\| I_{em} = 0 \tag{7}$$

where

$$I_{em} = \|I_E' \quad I_O' \vdots I_S' \quad I_M' \quad H'\|' \tag{8}$$

using the double transpose to save space.

The submatrix A_T has been shown to have an inverse (10), therefore a new matrix may be formed by multiplying A_m by this inverse. This matrix, called the node segregate matrix becomes

$$\begin{aligned} P &= A_T^{-1}\|A_T \vdots A_C\| \\ &= \|U \vdots A_T^{-1}A_C\|. \end{aligned} \tag{9}$$

The element order indicated by the first two current matrix entries in the right hand side of (8) suggests partitioning (9) so that

$$P = \left\| \begin{array}{cc:ccc} U_E & O & P_{1S} & P_{1M} & P_{1H} \\ O & U_O & P_{2S} & P_{2M} & P_{2H} \end{array} \right\|. \tag{10}$$

In the subscript notation of (8) and (10), E refers to ideal voltage sources, O to virtual elements, S to self-admittance elements, M to elements with both mutual and self admittance, and H to ideal current sources. The numbers 1 and 2 refer to submatrix rows in P.

The product of (8) and (10) is the segregate current equation

$$\left\| \begin{array}{cc:ccc} U_E & O & P_{1S} & P_{1M} & P_{1H} \\ O & U_O & P_{2S} & P_{2M} & P_{2H} \end{array} \right\| \left\| \begin{array}{c} I_E \\ I_O \\ \hdashline I_S \\ I_M \\ H \end{array} \right\| = 0. \tag{11}$$

Since the current matrix I_O is a null matrix, the following matrix equation is obtained from (11).

$$P_{2S}I_S + P_{2M}I_M + P_{2H}H = 0$$

or

$$\|P_{2S} \quad P_{2M}\| \left\| \begin{array}{c} I_S \\ I_M \end{array} \right\| + P_{2H}H = 0. \tag{12}$$

An admittance equation for the self and mutual elements may be written as

$$\left\| \begin{array}{c} I_S \\ I_M \end{array} \right\| = \left\| \begin{array}{cc} Y_S & O \\ O & Y_M \end{array} \right\| \left\| \begin{array}{c} V_S \\ V_M \end{array} \right\| \tag{13}$$

where Y_S is a diagonal matrix of self admittances and Y_M is a non-diagonal matrix of self and mutual admittances for the mutual elements.

A relation for the voltages across admittance elements in terms of node voltages is now needed. To obtain this relation, consider the equation

$$V_{em} = P'V_a \tag{14}$$

where V_a is a column matrix of $(n-1)$ entries representing an undefined set of voltage variables. Or in more detail,

$$\left\| \begin{array}{c} E \\ V_O \\ \hdashline V_S \\ V_M \\ V_H \end{array} \right\| = \left\| \begin{array}{cc} U_E & O \\ O & U_O \\ \hdashline P_{1S}' & P_{2S}' \\ P_{1M}' & P_{2M}' \\ P_{1H}' & P_{2H}' \end{array} \right\| V_a. \tag{15}$$

One of the relations obtainable from (15) is

$$\left\| \begin{array}{c} E \\ V_O \end{array} \right\| = \left\| \begin{array}{cc} U_E & O \\ O & U_O \end{array} \right\| V_a$$

which indicates that

$$V_n = \left\| \begin{matrix} E \\ V_O \end{matrix} \right\| = \left\| \begin{matrix} E \\ V_n \end{matrix} \right\|. \tag{16}$$

In the last expression, V_O has been replaced by the symbol V_n denoting the matrix of unknown node voltages. It should be noted that there are $(n - 1 - n_s)$ entries in V_n.

Using (16) in (15) yields the desired relation

$$\left\| \begin{matrix} V_S \\ V_M \end{matrix} \right\| = \left\| \begin{matrix} P_{1S}' & P_{2S}' \\ P_{1M}' & P_{2M}' \end{matrix} \right\| \left\| \begin{matrix} E \\ V_n \end{matrix} \right\|. \tag{17}$$

When matrix equations (12), (13) and (17) are combined, the result is

$$\|P_{2S} \quad P_{2M}\| \left\| \begin{matrix} Y_S & O \\ O & Y_M \end{matrix} \right\| \left\| \begin{matrix} P_{1S}' & P_{2S}' \\ P_{1M}' & P_{2M}' \end{matrix} \right\| \left\| \begin{matrix} E \\ V_n \end{matrix} \right\| + P_{2H}H = 0. \tag{18}$$

Finally, performing the matrix operations indicated in (18) yields

$$\|P_{2S}Y_SP_{1S}' + P_{2M}Y_MP_{2M}'\|E + \|P_{2S}Y_SP_{2S}' + P_{2M}Y_MP_{2M}'\|V_n + P_{2H}H = 0. \tag{19}$$

The only unknown in Eq. 19 is the node voltage matrix V_n containing $(n - 1 - n_s)$ unknown node voltage entries. The system of equations can be solved for the unknown node voltages provided that the inverse of the matrix multiplying V_n in Eq. 19 exists. Thus the entries in V_n represent a suitable set of independent node voltages. A non-trivial steady state solution of course requires that at least one of the entries in E or H be non-zero.

From the method of derivation, the node voltages are always such that the segregate current equations are satisfied; therefore Kirchhoff's current law is satisfied. However it is necessary to show that Kirchhoff's voltage law is also satisfied. That it is may be shown as follows.

An equation similar to (5) written for the augmented network is

$$B_m V_{em} = 0$$

or using Eq. 14,

$$B_m P' V_n = 0. \tag{20}$$

But the node segregate matrix P is

$$P = A_T^{-1} A_m$$

so that Eq. 20 becomes

$$B_m A_m' \|A_T^{-1}\|' V_n = 0. \tag{21}$$

It can be shown that the product $B_m A_m'$ is a null matrix (12). Thus Eq. 20 is identically satisfied. But Eq. 20 contains loop voltage equation (5) for the original network, thus the loop voltage equation and Kirchhoff's voltage law are both satisfied.

The satisfaction of both of the laws of Kirchhoff assures the validity of Eq. 19. Evidently this matrix equation represents the desired node system of equations.

FORMULATING THE NODE SEGREGATE MATRIX BY INSPECTION

It was previously indicated that segregate sets of elements could be formed on the basis of cuts through the elements of a network tree. This provides a convenient method for writing the node segregate matrix P so that it contains the unit submatrices indicated in Eq. 10. The method of forming the tree of ideal voltage source elements and virtual elements has been explained.

The procedure for formulating the P matrix is as follows.

1. Identify the $(e + n_0)$ columns in matrix P according to the element order in the current matrix of Eq. 11.
2. Identify the rows in matrix P according to the I_E and O elements using the same order as in the first $(n - 1)$ columns.
3. Let a cut through each of the tree elements in turn define a segregate set of elements and let the orientation of the cut element determine the orientation of the segregate set. There are $(n - 1)$ such segregate sets.
4. Let each segregate set determine the entries in one row of matrix P. Enter a $(+1)$ in a particular column if the element identifying that column is an element of the segregate, with relative orientation the same as that of the defining element, (-1) for the same case except opposite orientation, and zero if the column element is not an element of the segregate.

The submatrices in matrix P are evidently defined according to the element grouping in the rows and columns as indicated above and in Eq. 10. When one type of element is not present in a network, the corresponding submatrix entries in P and I_e may be omitted.

EXAMPLES USING MATRIX OPERATIONS

As a first example to demonstrate the methods thus far set forth, the node system of equations will be developed for a network based on Fig. 1. Let elements one, two and three be I_E elements; four, five, six, seven and eight be self-admittance elements; and let element nine be an I_H element. Node G is chosen as the reference node, and D and F as independent nodes.

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In view of the choice of independent nodes, the tree of I_E and O elements will take the form indicated in Fig. 2.

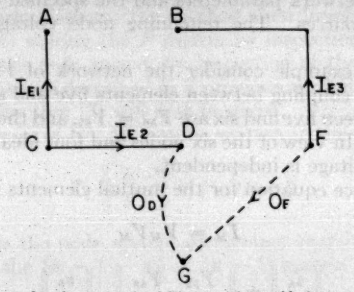

FIG. 2. A tree for the network of Fig. 1 with specified types of elements.

The segregate sets defined by this tree determine the following form for the P matrix. The submatrices P_{1M} and P_{2M} are omitted since the network involves no mutual admittances.

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ \\ D \\ F \end{array} \left\| \begin{array}{c|cc|cc|ccccccc} & 1 & 2 & 3 & D & F & 4 & 5 & 6 & 7 & 8 & 9 \\ \hline 1 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 3 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ \hline D & 0 & 0 & 0 & 1 & 0 & 1 & 1 & -1 & -1 & 0 & 1 \\ F & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 0 & 0 & -1 & -1 \end{array} \right\| = \left\| \begin{array}{cccc} U_E & O & P_{1S} & P_{1H} \\ O & U_O & P_{2S} & P_{2H} \end{array} \right\|.$$

The element defining each segregate is listed at the left-hand end of the appropriate segregate row of P.

The node system of equations may now be obtained by performing the matrix operations indicated in Eq. 19. The submatrices P_{1S}, P_{2S} and P_{2H} in the node segregate matrix may be easily identified, and in this example, submatrices P_{1M} and P_{2M} are null matrices. The row order in P, and the order of columns corresponding to self-admittance elements indicate that the voltage and self-admittance matrices are

$$E = \left\| \begin{array}{c} e_1 \\ e_2 \\ e_3 \end{array} \right\|, \quad V_n = \left\| \begin{array}{c} v_D \\ v_F \end{array} \right\|, \quad \text{and} \quad Y_S = \left\| \begin{array}{ccccc} Y_4 & 0 & 0 & 0 & 0 \\ 0 & Y_5 & 0 & 0 & 0 \\ 0 & 0 & Y_6 & 0 & 0 \\ 0 & 0 & 0 & Y_7 & 0 \\ 0 & 0 & 0 & 0 & Y_8 \end{array} \right\|.$$

The partitioning in P indicates that the current matrix H is a single row matrix, with the single entry h_9.

The indicated matrix operations yield the following node system of equations.

$$\left\| \begin{array}{ccc} -Y_4 & Y_4+Y_6 & Y_4+Y_8 \\ Y_4 & -Y_4 & -(Y_4+Y_8) \end{array} \right\| \left\| \begin{array}{c} e_1 \\ e_2 \\ e_3 \end{array} \right\| + \left\| \begin{array}{cc} Y_4+Y_5 & -(Y_4+Y_8) \\ +Y_6+Y_7 & \\ -(Y_4+Y_5) & Y_4+Y_8+Y_8 \end{array} \right\| \left\| \begin{array}{c} v_D \\ v_F \end{array} \right\| + \left\| \begin{array}{c} h_9 \\ -h_9 \end{array} \right\| = 0.$$

This node system of equations may be solved by any standard method. The independent node voltages v_D and v_F are thus determined in terms of the network parameters and the specified values associated with the ideal sources. The remaining node voltages can be easily determined.

As a second example consider the network of Fig. 3, containing mutual magnetic coupling between elements five and six. The mutual admittances between five and six are $Y_{56} = Y_{65}$, and the self admittances are Y_{55} and Y_{66}. In view of the six nodes and four ideal voltage sources, only one node voltage is independent.

The admittance equation for the mutual elements is

$$I_M = Y_M V_M$$

or

$$\left\| \begin{matrix} i_5 \\ i_6 \end{matrix} \right\| = \left\| \begin{matrix} Y_{55} & Y_{56} \\ Y_{65} & Y_{66} \end{matrix} \right\| \left\| \begin{matrix} v_5 \\ v_6 \end{matrix} \right\|. \tag{22}$$

Taking node G as the reference and v_B as the independent node voltage determines the tree made up of I_E elements one, two, three and four and the virtual element O_C. The P matrix takes the following form in which two of the submatrices are omitted in view of the absence of I_H elements.

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ \\ C \end{array} \begin{array}{c} \begin{array}{ccccccccccc} 1 & 2 & 3 & 4 & C & 7 & 8 & 9 & 5 & 6 \end{array} \\ \left\| \begin{array}{ccccc:ccccc} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 & 0 & 1 \end{array} \right\| \end{array} = \left\| \begin{matrix} U_E & O & P_{1S} & P_{1M} \\ O & U_O & P_{2S} & P_{2M} \end{matrix} \right\|.$$

The matrix operations of Eq. 19 now yield the single node equation

$$-Y_7 e_1 + Y_8 e_2 - Y_9 e_4 - (Y_7 + Y_{66} - Y_{65}) e_2 + (Y_7 + Y_8 + Y_9 + Y_{66}) v_C = 0. \tag{23}$$

This equation can easily be solved for the node voltage v_C after which this value together with the specified voltage values may be used to determine the remaining node voltages.

WRITING THE NODE SYSTEM BY INSPECTION

The node system of equations may in a sense be regarded as a restatement in different form, of information contained in the network diagram. It should therefore be possible to write the node system directly by inspection of the network diagram. Methods for doing this have in fact been commonly applied to the simpler types of networks (4).

In the case of more complex networks, particularly those in which

ideal voltage sources are not directly connected to the reference node, it is the opinion of the authors that the node system can be more accurately and easily written by inspection of the node segregate matrix P. Given the network diagram, the method then involves two straightforward steps: (1) writing the P matrix by inspection of the diagram, and (2) writing the node system by inspection of the P matrix. The first of these steps was described in a previous section.

A procedure for step (2) will now be given.

The node system of Eq. 19 may be written in the form

$$Y_n \left\| \begin{matrix} E \\ V_n \end{matrix} \right\| + P_{2H}H = 0 \tag{24}$$

where Y_n denotes the node system admittance matrix. From Eq. 19, this is evidently the $(n - 1 - n_s) \times (n - 1)$ matrix

$$Y_n^* = \|P_{2S}Y_SP_{1S}' + P_{2M}Y_MP_{1M}' \vdots P_{2S}Y_SP_{2S}' + P_{2M}Y_MP_{2M}'\|.$$

The entries in Y_n can be determined in an orderly manner if the $(n - 1)$ columns of Y_n are first identified in the same manner as the rows of the node segregate matrix P, and the $(n - 1 - n_s)$ rows of Y_n in the same manner as the last $(n - 1 - n_s)$ rows of P, which correspond to the independent node voltages. The rules for writing the node system of equations are as follows.

1. Denote the i^{th} row j^{th} column entry in Y_n by S_{ij}. Two cases should be considered.

(*a*) If $j = i$, $S_{ij} = S_{ii}$ is positive and is the sum of all self admittances with non-zero entries in row i of the node segregate matrix P.

(*b*) If $j \neq i$, S_{ij} is the algebraic sum of all self admittances with non-zero entries common to rows i and j in matrix P and the mutual admittances which couple elements in row j of P into elements in row i of P. The sign of an admittance term in S_{ij} is positive if the two entries in P corresponding to this term have the same sign and negative if they have opposite signs.

2. The entries in the column voltage matrix $\|E'V_n'\|'$ are identified in accordance with the rows of the node segregate matrix, and in the same order. This identification corresponds with that of the columns of Y_n, and also the first $(n - 1)$ columns of the node segregate matrix P.

3. The entry in row i of the term $P_{2H}H$ in Eq. 24 is the sum of the I_H element entries in the i^{th} row of P, each multiplied by its specified current h.

THE NODE SYSTEM OF EQUATIONS FOR A NETWORK CONTAINING VACUUM TUBES

As indicated previously, the small signal behavior of a vacuum tube may be specified by a matrix admittance equation (**5**). The self and

mutual admittances are dependent on the type of connection, that is whether the common cathode, common plate or common grid circuit is used. Given the admittances, these can be incorporated into the node system of equations in a straightforward manner.

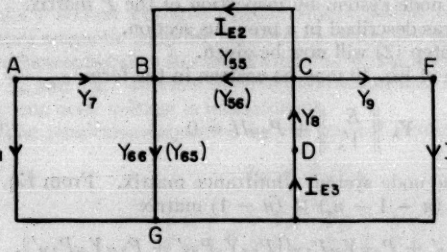

FIG. 3. Diagram of a network containing mutual magnetic coupling.

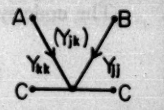

FIG. 4. Diagram representation of a vacuum tube for a case in which mutual admittance Y_{kj} is zero.

The subdiagram of Fig. 4 may be used to represent the tube in the network diagram. The symbols A and B represent input and output nodes which are connected to the common node C through the input and output self admittances Y_{kk} and Y_{jj}, respectively. The symbol Y_{jk} denotes mutual admittance between the input and output self-admittance elements. The admittance Y_{kj} is zero for low frequency operation in both the common cathode and common plate connections, but in general is not zero in the common grid case.

The matrix admittance equation corresponding to Fig. 4 in which $Y_{kj} = O$ is

$$\begin{Vmatrix} i_k \\ i_j \end{Vmatrix} = \begin{Vmatrix} Y_{kk} & O \\ Y_{jk} & Y_{jj} \end{Vmatrix} \begin{Vmatrix} v_k \\ v_j \end{Vmatrix}.$$

For the common cathode case in particular, where A becomes the grid, B the plate and C the cathode, the self and mutual admittances are

$Y_{kk} = Y_g$, the grid input self admittance,
$Y_{jj} = g_p$, the plate self conductance, and
$Y_{jk} = g_M$, the tube transconductance from grid to plate.

The following example shows that the node system of equations may be readily written for a network containing vacuum tubes. Such a network is shown in the diagram of Fig. 5, in which Y_8 may be recognized as a feedback element.

Choosing node E as the reference, the virtual elements needed in the tree of this network are O_B, O_C and O_D, assumed to be connected be-

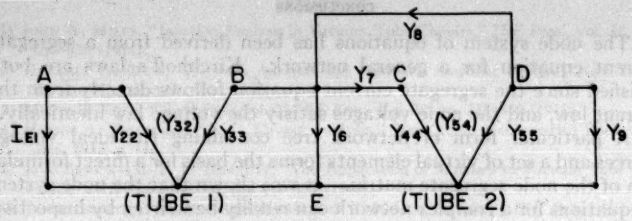

FIG. 5. Diagram of a network containing two vacuum tubes and a feedback element.

tween nodes B, C and D, respectively, and the reference. These virtual elements together with element I_{E1} comprise the network tree.

The node segregate matrix becomes

$$P = \begin{array}{c} \\ 1 \\ B \\ C \\ D \end{array} \left\| \begin{array}{c|cccc|cccc|cccc} & 1 & B & C & D & 6 & 7 & 8 & 9 & 2 & 3 & 4 & 5 \\ \hline 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \hline B & 0 & 1 & 0 & 0 & 1 & 1 & -1 & 0 & 0 & 1 & 0 & 0 \\ C & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 \\ D & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{array} \right\| = \left\| \begin{array}{cccc} U_E & O & P_{1S} & P_{1M} \\ O & U & P_{2S} & P_{2M} \end{array} \right\|.$$

The node system of equations obtained by inspection of the node segregate matrix and knowledge that the only mutual coupling is from element two into element three and from element four into element five is

$$\begin{array}{c} \\ B \\ C \\ D \end{array} \left\| \begin{array}{cccc} 1 & B & C & D \\ & Y_6 + Y_7 & & \\ Y_{32} & + Y_8 + Y_{33} & - Y_7 & - Y_8 \\ 0 & - Y_7 & Y_7 + Y_{44} & 0 \\ 0 & - Y_8 & Y_{54} & Y_8 + Y_9 + Y_{55} \end{array} \right\| \left\| \begin{array}{c} e_1 \\ v_B \\ v_C \\ v_D \end{array} \right\| = 0.$$

If the common cathode connection is used for each tube, the tube parameters become $Y_{33} = g_{p1}$, $Y_{32} = g_{M1}$, $Y_{44} = Y_{g2}$, $Y_{55} = g_{p2}$, and $Y_{54} = g_{M2}$.

The node system of equations then takes the form

$$\left\| \begin{array}{ccc} Y_6+Y_7+Y_8+g_{p1} & -Y_7 & -Y_8 \\ -Y_7 & Y_{g2}+Y_7 & 0 \\ -Y_8 & g_{M2} & Y_8+Y_9+g_{p2} \end{array} \right\| \left\| \begin{array}{c} v_B \\ v_C \\ v_D \end{array} \right\| = \left\| \begin{array}{c} -g_{M1}e_1 \\ 0 \\ 0 \end{array} \right\|.$$

It may be noted that the unilinear mutual coupling in tube two results in this example in a non-symmetrical matrix.

CONCLUSIONS

The node system of equations has been derived from a segregate current equation for a general network. Kirchhoff's laws are both satisfied since the segregate current equation follows directly from the current law, and the node voltages satisfy the voltage law identically.

A particular form of network tree containing the ideal voltage sources and a set of virtual elements forms the basis for a direct formulation of the node segregate matrix. It was shown that the node system of equations for a complex network can readily be written by inspection of its node segregate matrix, if the element characteristics are properly considered.

It is found to be convenient to represent devices containing mutual coupling by their admittance relations as a preliminary to forming the node system of equations. This type of representation eliminates the need for voltage or current generators within the small signal equivalent circuits of vacuum tubes and transistors.

The following definition is proposed as one result of this study:

The node system of equations is a set of linearly independent simultaneous equations describing the behavior of a network, in which the unknown variables are exclusively a set of independent node voltages of the network, and in which the number of equations and independent node voltages are each equal to the number of network nodes less the number of ideal voltage sources less one.

NOMENCLATURE

k = subscript denoting the k^{th} network element, node or variable.
I_E = ideal voltage source element.
n_e = number of ideal voltage sources in a network.
$e_k \equiv e_k(t)$ = specified voltage of element I_{Ek}.
I_H = ideal current source element.
n = number of nodes in a network.
$h_k \equiv h_k(t)$ = specified current of element I_{Hk}.
$Y_k \equiv Y_k(p)$ = admittance of the k^{th} element having self admittance only in mhos.
$Y_{kk} \equiv Y_{kk}(p)$ = self admittance of the k^{th} element having both self and mutual admittance, in mhos.
$Y_{kj} \equiv Y_{kj}(p)$ = mutual admittance coupling the j^{th} element into the k^{th} element, in mhos.
$Y_{jk} \equiv Y_{jk}(p)$ = mutual admittance coupling the k^{th} element into the j^{th} element, in mhos.
$v_k \equiv v_k(t)$ = voltage across the k^{th} element.
$i_k \equiv i_k(t)$ = current through the k^{th} element, in amperes.
e = number of elements in a network.
O_k = zero admittance (virtual) element connected between node k and the reference node.
$v_A \equiv v_A(t)$ = node voltage at node A = voltage across virtual element O_A.
n_O = number of type O (virtual) elements in an augmented network.
t = time in seconds.
$p \equiv d/dt$ = derivative operator.

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INSTABILITY OF A CONTINUOUSLY INHOMOGENEOUS VISCOELASTIC HALF-SPACE UNDER INITIAL STRESS

BY

M. A. BIOT [1]

ABSTRACT

Surface folding due to instability is analyzed for continuously inhomogeneous half-space of viscoelastic properties. The medium is under the combined action of a horizontal compression and gravity. The material is of uniform density. Its viscoelastic properties vary exponentially with depth so that it degenerates into a fluid at large depth. It is assumed that the deformations verify the thermodynamic theory of irreversible processes. Folding wavelengths are evaluated. The general correspondence principle as formulated earlier by the author shows the results to be immediately applicable to either viscoelastic or purely elastic media. A numerical application to the geophysical scale indicates that a continuous viscosity gradient resulting from the temperature variation with depth cannot account for orogenesis by the mechanism of instability in compression.

1. INTRODUCTION

In previous work (1, 2, 3)[2] we have examined the stability of inhomogeneous viscoelastic media under initial stress. All these problems dealt with a discontinuous system in which one homogeneous layer was embedded in another homogeneous medium or lying on top of the latter. The influence of gravity was also examined (4). Part of the initial stress in addition to the hydrostatic pressure is a compressive stress which produces an instability. This instability manifests itself by the growth of folds in the layer. The surface instability of a homogeneous half-space was also treated (3) as a particular case of a more general problem.

The present analysis deals with a half-space whose inhomogeneity is *continuously* distributed. The hydrostatic pressure due to the weight of the material is superposed on a horizontal compression.

The material is viscoelastic and incompressible with properties decaying exponentially with depth so that the medium becomes a fluid at large depth. The physical properties assumed are quite general but subject to the requirements of the thermodynamics of irreversible processes (6). The analysis covers the particular limiting cases of the inhomogeneous purely viscous solid and the inhomogeneous purely elastic medium for which the instability degenerates into a buckling. Section 2

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

presents the analytical formulation of the problem in which we have used equations for the deformation of a continuum under initial stress developed much earlier (7). Section 3 deals with the analytical solution and Section 4 with the numerical discussion. A numerical application to geophysics is also discussed indicating that large scale folding in the earth cannot be explained by the present mechanism on the basis of a continuous and isotropic viscosity gradient which itself is the result of the temperature increase with depth.

The results obtained here are equally applicable to either *viscoelastic* or *purely elastic media*. This is a consequence of the general *correspondence principle* as formulated and discussed by this writer in earlier work (6, 8, 10). The plot in Fig. 2, for example, applies immediately to the case of a purely elastic half-space whose rigidity modulus decreases exponentially with depth, and the buckling stress is given by Eq. 41.

Results of the experimental verification of the theory of folding of layered viscoelastic media by the use of models will be presented in later publications. A preliminary report on these tests has already been given (5). The task data show good agreement with the theory.

The author is indebted to A. Winzer for assistance in the analytical work and to A. S. Ginzbarg for the numerical solutions on the digital computer.

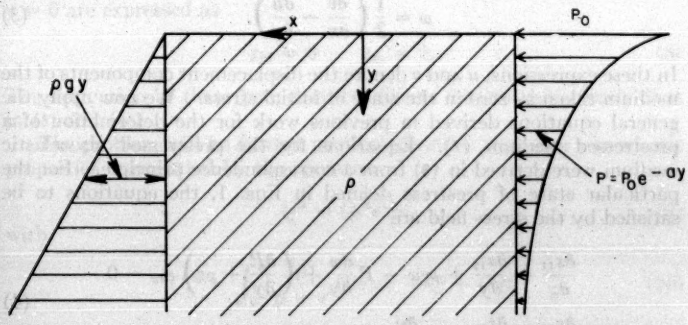

FIG. 1. Inhomogeneous viscoelastic half-space. The initial stress is the hydrostatic pressure $\rho g y$ and a horizontal compression P function of depth.

2. FORMULATION OF THE PROBLEM

Consider an inhomogeneous viscoelastic half-space. We shall assume that it is incompressible. This assumption is not essential and the methods used hereafter are applicable to compressible media. However, the assumption simplifies the algebra quite appreciably.

The x-axis lies along the surface of the half-space and the y-axis is directed downward (Fig. 1). Let this medium be in a state of initial stress which is the result of the gravity forces and a horizontal compression $P(y)$ function of the depth y. The state of initial stress is then represented by the components

$$\begin{aligned} S_{11} &= -P(y) - \rho g y \\ S_{22} &= -\rho g y \\ S_{12} &= 0 \end{aligned} \tag{1}$$

where ρ is the mass density of the medium chosen to be constant and g is the acceleration of gravity. For $P = 0$ the initial stress reduces to a hydrostatic pressure, $S_{11} = S_{22} = -\rho g y$.

In the following, we have used the strain components

$$e_{xx} = \frac{\partial u}{\partial x} \qquad e_{yy} = \frac{\partial v}{\partial y}$$
$$e_{xy} = \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) \tag{2}$$

and the rotation

$$\omega = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right). \tag{3}$$

In these expressions, u and v denote the displacement components of the medium taken as zero in the state of initial stress. We now apply the general equations derived in previous work for the deformation of a prestressed medium (**7**). Equations for the prestressed viscoelastic medium were derived in (**8**) from a correspondence principle. For the particular state of prestress defined in Eqs. 1, the equations to be satisfied by the stress field are

$$\begin{aligned} &\frac{\partial s_{11}}{\partial x} + \frac{\partial s_{12}}{\partial y} + \rho g \omega - P\frac{\partial \omega}{\partial y} + \left(\frac{\partial P}{\partial y} + \rho g\right) e_{xy} = 0 \\ &\frac{\partial s_{12}}{\partial x} + \frac{\partial s_{22}}{\partial y} - P\frac{\partial \omega}{\partial x} - \rho g e_{xx} = 0. \end{aligned} \tag{4}$$

The components s_{11}, s_{22} and s_{12} represent the incremental stress field with respect to axes rotated locally through an angle ω.

The physical properties of the material are represented by the stress-strain relations. With certain limitations which are discussed elsewhere (**2, 5**) and are not important in the present problem, the stress-strain relations for the incremented stresses are the same as for an

incompressible material in the absence of pre-stress. They are written

$$\begin{aligned} s_{11} - s &= 2Q^* e_{xx} \\ s_{22} - s &= 2Q^* e_{yy} \\ s_{12} &= 2Q^* e_{xy}. \end{aligned} \tag{5}$$

The left-hand side represents the stress deviator, that is, we put

$$\tfrac{1}{2}(s_{11} + s_{22}) = s. \tag{6}$$

The operator Q^* was derived (6) from the thermodynamics of irreversible processes and shown to be

$$Q^* = \int_0^\infty \frac{p}{p+r} Q(r)dr + Q + Q'p \tag{7}$$

with $p = \dfrac{d}{dt}$. In the present problem the material is assumed inhomogeneous with properties function of the depth y. This will be expressed by making the quantities $Q(r)$, Q and Q' functions of y in the operator Q^*.

The boundary condition results from the absence of forces on the free surface ($y = 0$). Using expressions for the boundary forces established in the earlier publications (3, 7) the boundary conditions at $y = 0$ are expressed as

$$s_{22} = 0 \qquad e_{xy} = 0. \tag{8}$$

The derivation of these boundary conditions follows very closely that in (3) and (4).

The problem will be further particularized by assuming viscoelastic properties represented by the operator

$$Q^* = Q_0^* e^{-\alpha y} \tag{9}$$

with

$$Q_0^* = \int_0^\infty \frac{p}{p+r} Q(r)dr + Q + Q'p. \tag{10}$$

This last operator is independent of y, and the inhomogeneity is represented only by the exponential factor in Q^*. This corresponds to viscoelastic properties which are nonhomogeneous but such that the relaxation spectrum is independent of the location. We used the term "homogeneous spectrum" to describe this type of viscoelasticity and have shown that it is associated with special properties as regards the stress distribution (8,9).

In a material represented by the operator (Eq. 9) the pre-stress

must also follow a similar law (Fig. 1). We write

$$P = P_0 \epsilon^{-\alpha y}. \tag{11}$$

Mathematically the problem reduces to solving Eqs. 4 after substituting the values of Eqs. 5 for the stresses. These are two equations for the three unknowns u, v, and s. A third equation is furnished by the condition of incompressibility.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{12}$$

The boundary conditions at $y = 0$ are given by Eqs. 8 and a further condition is that the solution must vanish at $y = \infty$.

3. ANALYTICAL SOLUTION

Following a procedure already used (4), it is possible to simplify the problem by eliminating gravity from the differential equations, provided the boundary conditions are appropriately modified. This is done by replacing the stress s_i by $s_{ij} - \rho v g \delta_{ij}$.[3] This substitution does not affect the stress-strain relations of Eqs. 5. On the other hand, if we take into account the condition of incompressibility (Eq. 12), Eqs. 4 become

$$\begin{aligned} \frac{\partial s_{11}}{\partial x} + \frac{\partial s_{12}}{\partial y} - P\frac{\partial \omega}{\partial y} + \frac{\partial P}{\partial y}\,\epsilon_{xy} &= 0 \\ \frac{\partial s_{12}}{\partial x} + \frac{\partial s_{22}}{\partial y} - P\frac{\partial \omega}{\partial x} &= 0. \end{aligned} \tag{13}$$

The boundary conditions (Eqs. 8) are replaced by

$$\begin{aligned} s_{22} &= \rho g v \\ e_{xy} &= 0. \end{aligned} \tag{14}$$

This is a particular application of a quite general procedure by which the hydrostatic stress is eliminated from the problem while a "buoyancy" term is added at the boundary.

The condition of incompressibility is satisfied by putting

$$\begin{aligned} u &= \frac{\partial \phi}{\partial y} \\ v &= -\frac{\partial \phi}{\partial x}. \end{aligned} \tag{15}$$

[3] δ_{ij} is the Kronecker symbol, that is, $\delta_{11} = \delta_{22} = 1$ and $\delta_{12} = 0$.

By substitution of the stresses (Eqs. 5) into Eqs. 13 and using expressions (15) for the displacements u and v, we arrive at two equations for the two unknowns ϕ and s. We further particularize the solution by looking for deformations which are sinusoidal along x. We therefore put

$$\begin{aligned} \phi &\equiv \varphi(y) \sin lx \\ s &= P_0 f(y) \epsilon^{-ay} \cos lx. \end{aligned} \tag{16}$$

The unknown functions φ and f must satisfy the ordinary differential equations

$$\begin{aligned} &\frac{1}{2}\frac{1+\zeta}{\zeta}(\varphi''' - \varphi' l^2 - a\varphi'' - a\varphi l^2) - lf = 0 \\ &\frac{1}{2}\frac{1-\zeta}{\zeta}(\varphi l^2 - \varphi'')l + \frac{1}{\zeta}a\varphi' l + f' - af = 0. \end{aligned} \tag{17}$$

The primes represent derivatives with respect to y and we have put

$$\zeta = \frac{P_0}{2Q_0^*}. \tag{18}$$

Solutions of Eqs. 17 are

$$\begin{aligned} \varphi &= Ae^{\beta_1 ay} + Be^{\beta_2 ay} \\ f &= Ce^{\beta_1 ay} + De^{\beta_2 ay}. \end{aligned} \tag{19}$$

Substitution of these expressions into Eqs. 17 leads to the characteristic equation for β,

$$[\beta(\beta^2 - \delta^2) - (\beta^2 + \delta^2)](\beta - 1) + k^2\delta^2(\delta^2 - \beta^2) + \frac{2\beta\delta^2}{1+\zeta} = 0 \tag{20}$$

where

$$k^2 = \frac{1-\zeta}{1+\zeta}$$

$$\delta = l/a.$$

There are two roots, β_1 and β_2, for β which have negative real parts. These two roots are used in the exponents of the solution expressed by Eqs. 19. This insures that the solution vanishes for $y = \infty$ (that is, at infinite depth). Equation 20 is a fourth degree equation in β. However, it is easily solved by noticing that it may be written as a second degree equation in $\beta(\beta - 1)$; for example, we may write

$$\zeta = \frac{\delta^4 - \delta^2[2\beta(\beta-1) - 1] + [\beta(\beta-1)]^2}{\delta^4 - \delta^2 - [\beta(\beta-1)]^2}. \tag{21}$$

Solving this quadratic equation for $\beta(\beta - 1)$ and again a second quadratic equation for β, we find

$$\left.\begin{matrix}\beta_1\\ \beta_2\end{matrix}\right\} = \frac{1}{2}\left[1 - \sqrt{1 + \frac{4\zeta}{1+\zeta}\left(\delta \pm \sqrt{\delta^2\zeta^2 - (1+\zeta)^2}\right)}\right]. \tag{22}$$

In the limiting case for $\delta \to \infty$, the roots reduce to

$$\begin{aligned}\beta_1 &= -\delta = -l/a\\ \beta_2 &= -k\delta = -kl/a.\end{aligned} \tag{23}$$

This case corresponds either to $a = 0$, that is, to the homogeneous half-space already examined (3) or to $l \to \infty$, that is, to a vanishing wavelength.

The four constants A, B, C, D in Eqs. 19 are not arbitrary. The relations among them are found by substitution in the differential equations. There are, therefore, only two arbitrary constants.

These two remaining constants are now eliminated by taking into account the boundary condition (Eq. 14) at the free surface ($y = 0$). This leads to the following characteristic equation:

$$\zeta = \frac{2\delta^2[\delta^2 - \beta_1\beta_2 - G(\beta_1 + \beta_2)]}{\beta_1^2\beta_2^2 - \delta^4 + \delta^2(\beta_1 + \beta_2)(\beta_1 + \beta_2 - 2G)} - 1. \tag{24}$$

The parameter

$$G = \frac{\rho g}{P_0 a} \tag{25}$$

represents the influence of gravity in non-dimensional form.

4. NUMERICAL RESULTS

The characteristic equation (24) may be considered as a relation between ζ and δ where G plays the role of a parameter. For a given value of G, this relationship represents the instability of the system as a function of the wavelength. This can easily be seen if we go back to the significance of the operator p as discussed in our earlier treatment of similar problems. A certain value of ζ corresponds to a certain positive value of p for which a time-dependent solution exists and is proportional to the increasing exponential e^{pt}. The value of p contained in ζ is a measure of the instability.

An important property may be stated here which is dependent on the nature of the operator Q_0^* itself. We have derived its form (Eq. 10) from the thermodynamics of irreversible processes. As pointed out in (6) and later papers, a consequence of the thermodynamics is the positive sign of all the terms in Eq. 10. Hence Q_0^* is a monotonically in-

creasing function of p. Therefore, considering the relationship between ζ and δ, a minimum of ζ corresponds to a maximum of p. For this value of δ the rate of growth of the deformation is a maximum. Wavelengths corresponding to this case will tend to appear more strongly than others. Following the terminology introduced in the earlier work we shall call this wavelength the dominant wavelength.

In solving the characteristic equation (24), we must distinguish cases for which the roots β_1 and β_2 are real or complex. From Eq. 22 it can be seen that real roots occur in the region

$$\delta > \frac{1}{\zeta} + 1 \tag{26}$$

and complex roots in the region defined by changing the direction of the inequality. For the case of complex roots, calculations are conveniently carried out by putting

$$\begin{aligned} R &= \left[1 + 20\delta^2 + 4\delta^2(1 + 4\delta^2)\left(\frac{1-\zeta}{1+\zeta}\right)\right]^{1/4} \\ \cos\frac{\theta}{2} &= \frac{1}{\sqrt{2}}\left[1 + \frac{1}{R^2}\left(1 + \frac{4\delta^2}{1+\zeta}\right)\right]^{1/2} \\ \beta_1 + \beta_2 &= 1 - R\cos\frac{\theta}{2} \\ \beta_1\beta_2 &= \frac{1}{4}\left[1 + R^2 - 2R\cos\frac{\theta}{2}\right]. \end{aligned} \tag{27}$$

The complex roots themselves are

$$\begin{aligned} \beta_1 &= \tfrac{1}{2}(1 - R\cos\tfrac{1}{2}\theta + iR\sin\tfrac{1}{2}\theta) \\ \beta_2 &= \tfrac{1}{2}(1 - R\cos\tfrac{1}{2}\theta - iR\sin\tfrac{1}{2}\theta). \end{aligned} \tag{28}$$

The characteristic equation (Eq. 24) was solved on an automatic computer for five values of the parameter G

$$G = 0,\quad \frac{1}{10},\quad \frac{1}{25},\quad \frac{1}{60},\quad \frac{1}{250}. \tag{29}$$

Plots of ζ *versus* δ are shown in Fig. 2.

The diagrams are plotted in the range $\delta < 1$. In this range the roots β_1 and β_2 are complex. The minimum value of ζ and the corresponding values of δ and β_1, β_2 are given in Table I.

The value δ_d of δ which corresponds to ζ_{min} yields the dominant wavelength,

$$L_d = \frac{2\pi}{a\delta_d}. \tag{30}$$

The value of δ_d for $G = 1/1000$ was evaluated by interpolation. In the absence of gravity ($G = 0$), the dominant wavelength goes to infinity ($\delta_d = 0$). It is found that δ_d is represented approximately by the formula:[4]

$$\delta_d = 2.2G^{6/10}. \tag{31}$$

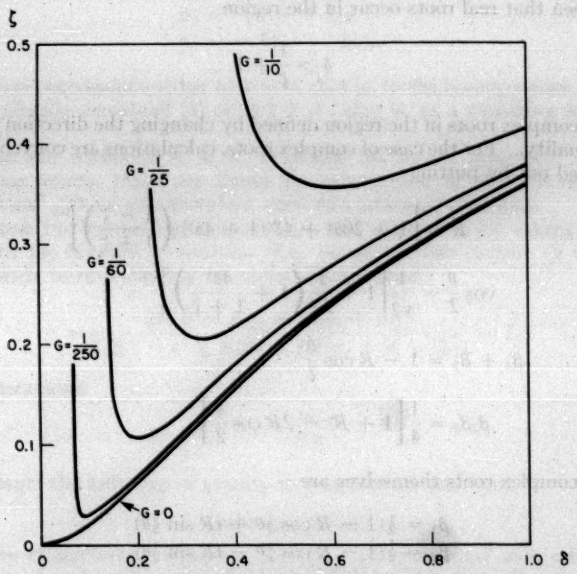

FIG. 2. The stability variable ζ as a function of the dimensionless wave-number δ and the gravity parameter G.

TABLE I.

G	ζ_{min}	δ_d	β_1, β_2
1/10	0.356	0.585	$-0.29 \pm 0.36i$
1/25	0.206	0.326	$-0.13 \pm 0.35i$
1/60	0.107	0.196	$-0.06 \pm 0.17i$
1/250	0.029	0.074	$-0.010 \pm 0.072i$
1/1000	...	0.035	...
0	0	0	...

[4] This result was reported earlier (5).

The value of ζ goes to infinity below a certain value of δ which depends on G. The physical significance of this corresponds to the existence of a cut-off wavelength above which no instability exists. The value of δ for this vertical asymptote is determined by the condition that the denominator in Eq. 24 vanishes. For vanishingly small δ, the location of the asymptote is given by

$$\delta = \sqrt{G}. \tag{32}$$

The limiting case of a vanishing wavelength ($l \to \infty$) is found by putting $\delta = \infty$ in Eq. 24. This equation becomes

$$\zeta^3 + 2\zeta^2 - 2 = 0. \tag{33}$$

The real positive root of this equation is

$$\zeta = 0.839. \tag{34}$$

Hence the curves in Fig. 2 all tend toward the same horizontal asymptote for $\delta \to \infty$, that is, for vanishing wavelength. In this limiting case the influence of gravity disappears and the value of ζ (Eq. 34) is the same as that obtained previously (3) for the case of an homogeneous half-space without gravity.

An interesting feature results from the complex nature of the roots β_1, β_2. This means that the displacements of the semi-infinite medium exhibit a *damped-oscillatory* character as a function of the depth y. This can be seen from Eqs. 19, which are linear combinations of functions of the type

$$\exp\left[ay \operatorname{Re} \beta_1\right] \begin{matrix}\sin\\ \cos\end{matrix} \left[ay \operatorname{Im} \beta_1\right]. \tag{35}$$

The real and imaginary parts $\operatorname{Re} \beta_1$ and $\operatorname{Im} \beta_1$ of β_1 are shown in Table I. For example, for $G = 1/250$ the distribution with depth becomes

$$\exp\left[-0.01\, ay\right] \begin{matrix}\sin\\ \cos\end{matrix} \left[0.072\, ay\right]. \tag{36}$$

Over one wavelength the amplitude decays by a factor 0.418. The values of Table I indicate that the decay decreases with decreasing values of G. In solving the problem no restriction has been put on the nature of the operator Q_0^* except that it satisfies thermodynamic principles. This insures the existence of one positive root p corresponding to any value of ζ derived from the stability diagram (Fig. 2). In particular, the theory is applicable to a purely viscous medium. In this case the operator is

$$Q_0^* = \eta_0 p. \tag{37}$$

The viscosity η of this medium decreases exponentially with depth according to the equation

$$\eta = \eta_0 e^{-\alpha y}. \tag{38}$$

The theory also applies to a purely elastic material. In this case

$$Q_0^* = \mu_0. \tag{39}$$

The medium is an inhomogeneous elastic half-space whose rigidity modulus μ varies with depth according to the equation

$$\mu = \mu_0 e^{-\alpha y}. \tag{40}$$

The minimum value $\zeta_{\min}$ yields the *buckling load* P_0 by

$$P_0 = 2\mu_0 \zeta_{\min}. \tag{41}$$

The dominant wavelength does not depend on the particular nature of the viscoelastic material. For instance, its value for the viscous solid is the same as the buckling wavelength for the elastic case. The difference lies in the fact that folding always occurs in the purely viscous case in contrast with the elastic case where a minimum compression (Eq. 41) is required for buckling to appear.

In applications it is of interest to consider the magnitude of the folding. As pointed out above, the amplification of waves after a time t is e^{pt}. If we assume a viscous medium, the time required for it to shorten by 25 per cent at the surface is

$$t_1 = \frac{\eta_0}{P_0} \tag{42}$$

and the amplification of any initial wave at that time is e^{pt_1}. If we want this amplification to be equal to 100 then we must have

$$pt_1 = \log_e 100 = 4.60. \tag{43}$$

Substituting for p its expression in terms of ζ by eqs. 18 and 37, we derive

$$\zeta = \frac{1}{9.2}. \tag{44}$$

Hence this is the value of ζ required for the amplification to be 100 during a compressive deformation of 25 per cent. We see from Table I that for the dominant wavelength this requires that the value of G be about

$$G = \frac{1}{60}. \tag{45}$$

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Smaller values of G will generate higher amplifications. In application to geophysics we may put P_0 equal to a value of the order of the crushing strength of rock at the surface, for example, $P_0 = 10^9$ dynes/cm^2. With $\rho = 2.5$ and $G = 1/60$ we derive

$$\frac{1}{a} \cong 65 \text{ meters.} \tag{46}$$

This is the depth in which the viscosity decreases by a factor $1/e$.

The dominant wavelength for this case is found from Table I and Eq. 30. Its value is $L_d = 2100$ meters. The viscosity gradient required for such significant folding to happen and which is characterized by the value (46) of $1/a$ is much higher than that which results from increases of temperature with depth in the earth.

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CYCLES IN LOGICAL NETS*

BY

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ABSTRACT

This paper investigates the influence of cycles in a logical net upon the complexity of its behavior. The investigation is mainly concerned with two questions:

1. A logical net with a periodic input sequence produces a periodic output sequence; how is the spectrum of periodic outputs related to the level of cycle complexity?

2. Is there a level of complexity c (suitably defined) such that any behavior possible for a fixed logical net can be realized by a logical net constructed only of cycles of complexity $c' \leq c$? The first and more difficult question is fully answered only in the case of nets constructed of cycles having a feedback coefficient $r = 1$ (suitably defined). The second question is answered in the negative for individual cycles and it is conjectured that a similar answer holds for nets in general.

1. INTRODUCTION

Cycles in a logical net in general have a profound effect upon the net's behavior. This paper investigates the relation between the complexity of such cycles and the complexity of the behavior that results. Much of the investigation is concerned with two questions:

1. A logical net with a periodic input sequence produces a periodic output sequence; how is the spectrum of periodic outputs related to the level of cycle complexity?

2. Is there a level of complexity c (suitably defined) such that any behavior possible for a fixed logical net can be realized by a logical net constructed only of cycles of complexity $c' \leq c$?

The first and more difficult question is fully answered only in the case of nets constructed of cycles having a feedback coefficient $r = 1$ (suitably defined); Theorem 3 provides the answer for this case. The final theorem of the paper leaves the second question still unanswered for nets in general, but provides the following answer for individual cycles: If we consider the set C of all cycles having $n' \leq n$ delays and a feedback coefficient $r' < r$, then there is a cycle having n delays, a feedback coefficient r, and a state transition diagram which cannot be

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realized by any cycle in C. Equating cycle complexity to the number pair (n', r'), and defining an ordering (a lattice) such that $(n', r') < (n, r)$ just in case $(n' < n,\ r' \leq r)$ or $(n' \leq n,\ r' < r)$, we can answer the second question in the negative for individual cycles.

The theorem last mentioned rests upon an interlocked sequence of definitions, operations, and theorems leading from the simplest input-free cycles to arbitrary cycles with input. The operations in particular play a key part in the development; they provide a means of obtaining information about state cycles in the state transition diagram of an arbitrary net cycle in terms of the transition diagrams of simple input-free cycles. Aside from this role, these operations have application in a wide range of problems and questions concerning the relation of cycle complexity to complexity of behavior. Two simple examples of such applications are given at convenient places in the development; one application is to a conjecture of Burks and Wang (1)[2] and the other is to a problem investigated by de Bruijn (3).

The development proceeds in the following stages:

Section 2 defines the term "cycle" as used in this paper and shows the existence of a normal form for cycles. Intuitively, a cycle with n delays has a normal form in which there are also exactly n switches, with the output of the j^{th} delay going to the j^{th} switch (and perhaps others if the cycle is complex) which in turn acts as input to the $(j + 1)^{th}$ delay (modulo n). This normal form later makes possible a relatively simple definition of the order of a net cycle (the feedback coefficient mentioned earlier) and considerably simplifies the proofs by drastically reducing the number of cases and variations to be considered.

Section 3 considers the effect of periodic input upon the behavior of nets in which all cycles are of order 1 (each delay in the normal form of each cycle acts as input to exactly one switch in that cycle). This part of the investigation was motivated by the following consideration: If it could be shown that cycles only serve to provide logical nets with memories of various recycling times, then it would be possible to accomplish this function by means of cycles of order 1 ("simple cycles"). Under such conditions there would be little need to consider cycle complexity further except insofar as to show how to reduce the complex cycle to an equivalent simple cycle. In fact this conjecture proves to be untrue as is shown in a Corollary to Theorem 3. Theorem 3, besides giving a complete description of the output spectra for nets composed of simple cycles, shows that no such net can realize the behavior of a 2-delay cycle of order 2. Section 3 concludes by showing that a particular case of a conjecture of Burks and Wang (1) is true.

The sequence of theorems in Section 4 prepares the way for the results given in the last section. Section 4 begins by considering input-

[2] The boldface numbers in parentheses refer to the references appended to this paper.

free simple cycles with one switch and proceeds step-by-step to the general input-free cycle. At each level an operation is developed which yields information about the state transition diagrams of cycles at the next level in terms of properties established for the given level. That is, each theorem (with the exception of Theorem 8) provides information which when combined with the appropriate operation yields information appropriate to the next step. In Section 5 the results for input-free cycles are extended to the case of the general cycle with input, yielding the result (Theorem 12) mentioned earlier. This interlocking sequence of definitions and operations leading from input-free simple cycles to the general cycle with input can be summarized as shown in Table I (the reader is referred to context for exact definitions).

TABLE I.

Type of Net Cycle	Definitions Providing for Extension to Next Level	Operations Generating Extension to Next Level
(1) simple cycle without input	locally balanced switch; derived transition table; normal state cycle	inversion
(2) locally balanced cycle	order of switch	unbalancing
(3) input-free net cycle with one switch	normal form of net cycle; derived transition table for normal form	finite induction on number of switches in normal form of net cycle (listing possible effects of added switches on derived transition table)
(4) general input-free net cycle	constant input subgraphs, G_{I_i}, of transition graph G	selection operation of input-state $I(t)$ (selects $G_{I(t)}$ from G).
(5) general net cycle with input	rank of net-cycle	cascading of net cycles and net-cycle nets
(6) logical nets in general		

Because of the way in which a net cycle is defined, each element in a net can belong to at most one net cycle. It follows from this that the cycles in a logical net can be ranked and therefore that the cascading operation is sufficient to generate any logical net (cf. Burks-Wang (1) and Section 2 of the present paper). Hence, row (6) was added to the table although the present paper does not specifically consider the case (except for simple cycles in Section 3).

In Table I, the class of all net cycles of a given type properly includes preceding types of net cycle. Generally, the operations presented in the table and discussed throughout the paper are useful

in computing the behavior of particular net cycles as well as in proving theorems about the various types of net cycle.

In Section 4, Theorem 8 provides a constructive solution in terms of input-free locally balanced cycles of a problem investigated by de Bruijn (3). The proof illustrates the use of the operations just summarized in exploring questions other than the two posed at the outset. In Section 5, preceding Theorem 12, a method is given for computing the periodic input/periodic output relation of any given backwards deterministic cycle. The computation makes use of the properties of a related set of input-free locally balanced cycles. The paper concludes with a conjecture that the result of the final theorem holds for nets in general; that is, it is conjectured that the answer to the second question asked earlier is negative.

2. UNDERLYING CONCEPTS

The logical nets considered in this paper will be well-formed logical nets, essentially as defined by Burks and Wang in Part 2.3 of their paper (1). The nets are based upon a set of primitives consisting of a delay element and an infinite class of switching elements. Terminology, except where explicit definitions are given, will be that of Burks and Wang. The state transition graph will be used as a means of compactly picturing the behavior of a logical net. E. F. Moore's conventions (6) will be used except that the output associated with each net state will not in general be indicated.

As indicated in the introduction, the purpose of this paper will be to investigate the role of cycles in logical nets. The exact definition of cycle proceeds from the concept of one element of a net driving another. An element E_1 *directly drives* an element E_2, $E_1 \underline{\mathrm{d}} E_2$, if and only if the output of E_1 is connected to one of the inputs of E_2. A sequence of elements $F_1, \cdots, F_n$ is a *drive sequence from* F_1 *to* F_n if and only if $F_j \underline{\mathrm{d}} F_{j+1}$ for $j = 1, \cdots, n-1$. An element E_1 *drives* an element E_2, $E_1 \underline{\mathrm{D}} E_2$, if and only if there is a drive sequence from E_1 to E_2. Now, an element E_1 belongs to a cycle if and only if $E_1 \underline{\mathrm{D}} E_1$. This cycle consists of the set of all elements, E_j, such that both $E_1 \underline{\mathrm{D}} E_j$ and $E_j \underline{\mathrm{D}} E_1$. Or, more formally, a set of elements, C, is a *cycle* if and only if, for all E_i, E_j in C, (1) $E_i \underline{\mathrm{D}} E_j$ and $E_j \underline{\mathrm{D}} E_i$ and (2) no element of the net not in C satisfies condition (1). A set of elements, C', is a *subcycle* if and only if, for all E_i, E_j in C', (1) $E_i \underline{\mathrm{D}} E_j$ and $E_j \underline{\mathrm{D}} E_i$ and (2) for each relation $E_i \underline{\mathrm{D}} E_j$ of (1) all elements of the defining drive sequence are elements of C'. It is an immediate consequence of these definitions that each element of a net belongs to at most one cycle, although it may belong to several subcycles.

Note that the cycles in a well-formed net can be ranked as follows: A cycle is of *rank* 0 if none of its inputs is driven by an element of another cycle. A cycle is of *rank r* if at least one of its inputs is driven

by an element of another cycle of rank $r - 1$ and none of its inputs is driven by elements of other cycles of rank greater than $r - 1$.

Any cycle can be reduced to a much simpler *normal form*: if the cycle has n delays then there is an equivalent normal form which has at most n switches, so arranged that the output of delay j goes to switch

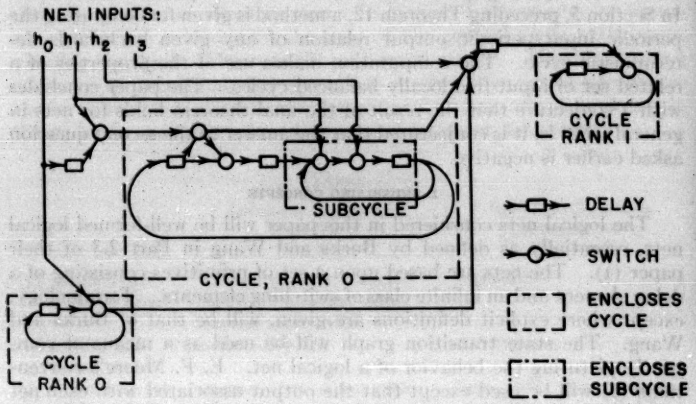


FIG. 1. A logical net with cycles marked.

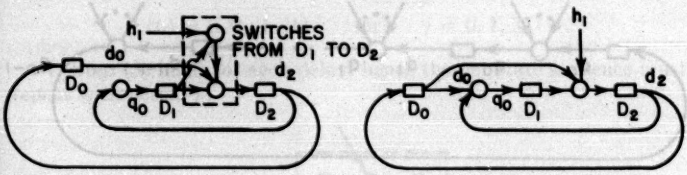
j (and perhaps others) and the output of switch j acts as input to delay $j + 1$ (modulo n). To establish this normal form the following definitions are required: A drive sequence in which all elements, other than the first and last, are switches will be called a *drive sequence of switches*. If there is at least one drive sequence of switches from an element E_1 to an element E_2 it will be said that E_1 *drives* E_2 *via switches*, $E_1 \underline{s} E_2$. The set of all switches belonging to drive sequences of switches from E_1 to E_2 will be called *the set of switches from* E_1 *to* E_2.

Let the set of delays belonging to a given cycle be ordered and labelled $D_0, D_1, \cdots, D_{n-1}$. Since we are dealing with well-formed nets and since each delay has but one input, there must be at most one switch S_j which directly drives delay D_j. If $D_{j-1(\mathrm{mod}\ n)} \not\underline{s} D_j$ simply add a new input to S_j with the proviso that the output of the switch is to be independent of this new input (that is, the output of the switch is still uniquely determined by the inputs initially given). By connecting the output of $D_{j-1(\mathrm{mod}\ n)}$ to this new input we have formally the result $D_{j-1(\mathrm{mod}\ n)} \underline{s} D_j$. Let this be done for all j, $j = 0, 1, \cdots, n - 1$.

For each j, consider *all* D_i in the cycle such that $D_i \underline{s} D_j$ (one such D_i will now be $D_{j-1(\mathrm{mod}\ n)}$). Among all the switches belonging to one or more of the sets of switches from the $\{D_i\}$ to D_j there will be a total

of k switch inputs not identified with the output of any other switch in these sets or with the outputs of the $\{D_i\}$. It can easily be seen that each assignment of states to these k switch inputs uniquely determines the input state of D_j when the output state of each D_i driving D_j via switches is given. Thus, assuming that there are m such D_i, we can replace all of the aforementioned switches by a single $(k + m)$ input switch. Of the inputs to this new switch, k will be identified as inputs to the cycle corresponding to the k selected inputs of the given switches. The other m inputs will be connected to the outputs of the m delays D_i. The output of the new switch will be connected to the input of D_j. Once this is done for each D_j, the normal form of the cycle results.

It is an immediate consequence of the construction method that the normal form of any cycle with n delays will have no more than n switches. Furthermore, each *cycle input* (the net inputs of a cycle when it is taken as the whole net) of the original cycle will drive via a switch exactly the same delays in the normal form as in the original form. Finally, at any time t, the input and output state of each delay in the normal form will be that of the same delay in the original form.



d_2	q_0
0	δ_0
1	δ_1

$q_0(t) \equiv f_0(d_2(t))$

d_0	d_2	q_0
0	0	δ_0
0	1	δ_1
1	0	δ_0
1	1	δ_1

$q_0(t) \equiv f_0(d_2(t))$

FIG. 2. A cycle and its normal form.

3. SIMPLE CYCLES

A hint of the role of cycles in logical nets comes from the following observation: For a net with n delays and no cycles, the net state at time t is totally determined by the sequence of input states from $t - n$ to $t - 1$—the net state at time t is completely independent of any net

state preceding time $t - n + 1$. Thus a net without cycles can only record the detection of an input event for at most n time-steps. In other words, if a logical net is to have "memory" or storage it must include cycles.

Upon noting the function of cycles as memory elements, one of the first questions which presents itself is: Can the full range of logical net behavior be obtained from nets using cycles of limited complexity? More precisely, in the class of well formed nets, is there a proper subset, defined in terms of some limitation on the cycles allowed, which can exhibit the full range of logical net behavior? It is fairly obvious that the number of cycles cannot be limited; this would contradict the existence of nets with arbitrarily large numbers of memory units. A possible first step would be to consider nets using only cycles with minimal feedback, that is, cycles with no proper subcycles. A cycle of this type, which I will call a *simple cycle*, can be more directly defined as a cycle in which each delay drives via switches exactly one other delay in the cycle. The normal form of such cycles is particularly simple (see Fig. 3).

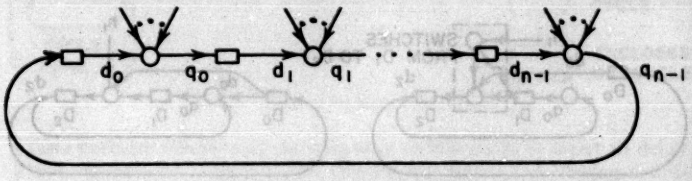


FIG. 3. Normal form of simple cycle.

The conjecture, then, with respect to simple cycles would be that for each logical net there is a net with, at most, simple cycles which has the same behavior. The conjecture is plausible on the view that the simple cycles provide "delay line" or "reverberatory" storage of various periods while the rest of the net provides encoding, switching, decoding, and other logical operations. The following three theorems show that this conjecture is in fact false. The last theorem of the section, Theorem 3, shows a good deal more than this— for any net composed of simple cycles, it characterizes the spectrum of net state periods resulting from periodic input sequences.

Theorem 1. If the sequence of input states of a simple cycle has a proper period m and the simple cycle has n delay elements, then the sequence of net states of the cycle has a proper period 2 l.c.m. (m, n) or a divisor thereof.

Proof outline[3]

1. Consider a particular delay, i. Let $d_i(t)$ be the state of this delay at time t. If $d_i(t_1) = d_i(t_2)$ and if the input sequence $I(t_1 + 1)$, $I(t_1 + 2), \cdots, I(t_1 + k)$ is identical to the input sequence $I(t_2 + 1)$, $\cdots$, $I(t_2 + k)$ for some $k = jn$, then $d_i(t_1 + k) = d_i(t_2 + k)$. (Consideration shows that this statement is not true in general for $k \neq jn$.) Under the same conditions if $d_i(t_1) = 1 - d_i(t_2)$ then $d_i(t_1 + k) = 1 - d_i(t_2 + k)$.

2. The input sequence repeats every m steps by hypothesis. Thus, for $k = jn$, the input sequence $I(t_1 + 1), \cdots, I(t_1 + k)$ is identical to the sequence $I(t_1 + k + 1), \cdots, I(t_1 + 2k)$ for j such that $k = jn$ = l.c.m. (m, n).

3. Since $d_i(t)$ can take on only two values, we must have by (1) and (2) either

$$d_i(t_1) = d_i(t_1 + \text{l.c.m. } (m, n))$$

or

$$d_i(t_1) = 1 - d_i(t + \text{l.c.m. } (m, n)) = d_i(t + 2 \text{ l.c.m. } (m, n)).$$

Thus, for $p = 2$ l.c.m. (m, n), we must have

$$d_i(t_1) = d_i(t_1 + jp) \qquad \text{for} \qquad j = 0, 1, 2, \cdots.$$

4. Step (3) holds for each delay, hence the net state sequence must repeat with period p.

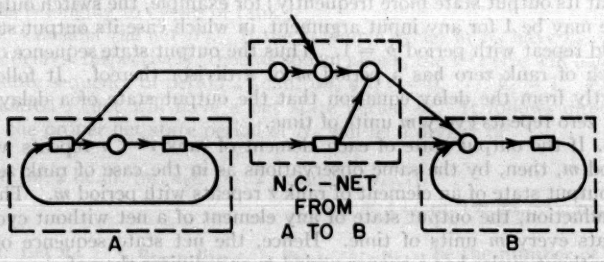


FIG. 4. An n.c.-net.

In nets constructed with more than one simple cycle, the simple cycles can be separated by subnets having no cycles. The following definitions are intended to give a precise interpretation to this statement: A drive sequence from an element F_1 to an element F_2 will be

[3] At the suggestion of the JOURNAL reviewers, the proofs of this theorem and of Theorems 5, 7, 8, 9, 10 and 11 have been omitted. They may be found in the microfilm edition of the author's thesis, "Cycles in Logical Nets," Ph.D. dissertation, 1959, The University of Michigan.

called an *n.c.-drive sequence* if none of the elements in the sequence other than the first and last, belongs to a cycle. A set of elements, A, *n.c.-drives* a set of elements, B, if there is at least one n.c.-drive sequence from some element of A to some element of B. The net consisting of all elements, other than elements of A and B, belonging to n.c.-drive sequences from A to B is the *n.c.-net from A to B*.

Theorem 2. If the sequence of input states to a net without cycles (for example, an n.c.-net) has a proper period m, then the net state sequence of the net has a proper period m or a divisor thereof.

Proof:

1. In order to simplify the proof let each element in the net without cycles be given a rank as follows: (1.1) if all the inputs of the element are net inputs of the given net, its rank is zero; (1.2) if an input of the element is identified with the output of an element of rank $r - 1$, and no input is identified with the output of an element of rank higher than $r - 1$, then the element is of rank r. Since an n.c.-net contains no cycles each element has a unique rank.

2. Now, the states of the inputs of each element of rank zero must repeat with period m since these states are merely components of the net input state. Therefore, since the output state of a switch is uniquely determined by the set of input states, the output state of each switch of rank zero repeats every m units of time. The switch may, of course, repeat its output state more frequently; for example, the switch output state may be 1 for any input argument, in which case its output state would repeat with period $p = 1$. Thus the output state sequence of a switch of rank zero has a period m or a divisor thereof. It follows directly from the delay equation that the output state of a delay of rank zero repeats every m units of time.

3. If the output state of each element of rank $r' < r$ repeats with period m, then, by the same observations as in the case of rank zero, the output state of an element of rank r repeats with period m. Thus, by induction, the output state of any element of a net without cycles repeats every m units of time. Hence, the net state sequence of a net without cycles has a proper period m or a divisor thereof.

We can now proceed to the general case with respect to the conjecture mentioned at the outset of this section: the case of nets in which all the cycles are simple cycles.

Theorem 3. A net in which all of the cycles are simple, having $n_1, \cdots, n_k$ delay elements, respectively, with a sequence of input states of proper period m, will have a net state sequence of period 2^{r_0+1} l.c.m. $(m, n_1, \cdots, n_k)$, where r_0 is the maximum of the cycle ranks.

Proof:

1. To begin with, note that every element of a net, N, is driven by one or more of the net inputs of N, except those elements belonging to a cycle having no cycle inputs (that is, the cycle, considered as a net, has no net inputs).

2. Let A_0 be the set of elements satisfying the following two conditions: (2.1) at least one input of the element is a net input of N, (2.2) the element does not belong to a cycle. Let C be the set of elements belonging to a given cycle of rank 0 (see Section 2). Any element driving C but not in C must be driven by an element of A_0 because C is of rank 0 and hence no element belonging to a cycle can drive an element of C. Let P be the net consisting of the n.c.-net from A_0 to C together with the elements of A_0. The net includes all elements of N driving cycle inputs of C. Since the net inputs of P are just the net inputs of N, the net state sequence of P must be of period m by Theorem 2. Thus the input state sequence of the cycle inputs to C must be of period m. Since C is a simple cycle the results of Theorem 1 apply—the net state sequence of C has a proper period 2 l.c.m. (m, n), or a divisor thereof, where n is the number of delays belonging to C.

3. Now, define inductively a set of nets N_j, for $j = -1, 0, 1, \cdots, r_0$, where r_0 is the maximal rank of the cycles in the net N. The net N_j consists of the following elements with their inputs identified as in N: (3.1) all elements of N_{j-1} (where N_{-1} is the set A_0); (3.2) all elements belonging to n.c.-nets from N_{j-1} to cycles of rank j; and (3.3) all elements belonging to cycles of rank j.

4. Consider the net N_0. If the k_0 simple cycles in N_0 have $n_0, \cdots, n_{k_0-1}$ delays, respectively, then as shown above, the i^{th} cycle will have a net state sequence of period 2 l.c.m. (m, n_i), $i = 0, \cdots, k_0 - 1$. Each associated P net will have a net state period m. It follows directly that the proper net state period of N_0 will be a divisor of

$$\text{l.c.m. } (m, 2 \text{ l.c.m. } (m, n_0), \cdots, 2 \text{ l.c.m. } (m, n_{k_0-1})) = 2 \text{ l.c.m. } (m, n_0, \cdots, n_{k_0-1}).$$

5. Let $n_0, \cdots, n_{k_0-1}, n_{k_0}, \cdots, n_{k_1-1}, \cdots, n_{k_j-1}$ be the number of delays belonging, in order, to each of the simple cycles from rank 0 through rank j.

6. Assume the net N_{j-1} has a net state sequence of period $p = 2^i$ l.c.m. $(m, n_0, \cdots, n_{k_{j-1}-1})$. By substituting N_{j-1} for A_0 and p for m in step (2) we see that a cycle C_i of rank j must have a net state sequence of period

$$2 \text{ l.c.m. } (p, n_i) = 2 \text{ l.c.m. } (2^i \text{ l.c.m. } (m, n_0, \cdots, n_{k_{j-1}-1}), n_i) = 2^{i+1} \text{ l.c.m. } (m, n_0, \cdots, n_{k_{j-1}-1}, n_i).$$

Applying the reasoning of step (4) we see then that the proper net state period of N_j will be a divisor of l.c.m. $(m, 2^{r+1}$ l.c.m. $(m, n_0, \cdots, n_{k_{j-1}})$.

7. Thus by induction on j, the net N, when the input states repeat with period m, will have a proper net state period which is a divisor of $p = 2^{r_0+1}$ l.c.m. $(m, n_0, \cdots, n_k)$. Again, for reasons similar to those noted at the end of Theorem 1, there exist combinations of switching elements and cycles giving N a net state sequence of proper period p.

Corollary. Let it be required that a net be in a chosen net state S_0 if and only if the number of occurrences, p, of a distinguished input state I_0 satisfies the equation $p \equiv 0 \bmod j$. (Simply, the net is required to "count," modulo j, the occurrences of input state I_0.) For nets in which all cycles are simple, j must equal 2^b for some positive integer b. (Such nets can only "count" modulo a power of 2.)

Proof:

Let the distinguished input state repeat with a proper period m. Then, since the net is to be in a unique net state S_0 for each j occurrences of the distinguished input state, it must have a net state which repeats with proper period $p = jm$ for all m. Or, by Theorem 3,

$$p = jm = 2^{r_0+1} \text{ l.c.m. } (m, n_0, \cdots, n_k).$$

This equation can only hold for all m if $j = 2^{r_0+1}$, since if m is chosen equal to k(l.c.m. $(n_0, \cdots, n_k)$) the above equation reduces to $jm = 2^{r_0+1}m$.

The corollary shows that no net composed of simple cycles can, for example, act as a ternary (base 3) counter. To do so it would have to be in some distinguished state S_0 for every third occurrence of the distinguished input state I_0. This condition holds *a fortiori* for a periodic input sequence, whence we would require $p \equiv 0 \bmod 3$, contradicting the corollary. Since there exists a 2-delay logical net which can count base 3, the corollary at once establishes the falsity of the conjecture stated at the beginning of this section.

If this result is not surprising, it at least shows the oversimplification present in the idea that the main function of cycles in a system is to provide "memory" or storage of information. Here we have systems with any number of cycles of arbitrary lengths (arbitrary recycling times) which have a very limited range of behavior, not because we restrict the complexity of the switching elements used, but because the cycles are limited in the complexity of their feedback patterns.

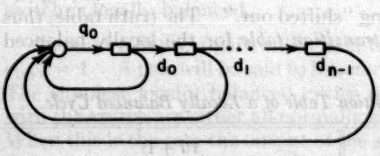
The results here also show that a particular case of a conjecture of Burks and Wang (1, p. 292) is true. The conjecture is: For any degree d, there is some transformation not realized by any net of degree d—a net is of degree d if it contains at least one cycle of degree d and none

of higher degree; a cycle is of degree d if it contains d delays. Since nets of degree 1 must be composed of simple cycles, we see that there are transformations on periodic input-state sequences not accomplished by any net of degree 1. By using periodic input as a tool one can often prove theorems concerning a given class of nets which would be difficult to prove in any other way.

4. INPUT-FREE CYCLES

The results of Section 3 show the behavior of a logical net to be severely restricted if the complexity of the net cycles is sufficiently limited. Moreover, the limitation on complexity need not concern the number of cycles or the number of delays in a cycle, but only the number of feedback loops (subcycles) per cycle. The effect of increasing the number of feedback loops in a cycle thus becomes a salient point of the study of cycles in logical nets.

The present section will start out by relating properties of the state-transition graph to changes in feedback in a class of input-free cycles called locally balanced cycles. Just as cycles are important features of net structure, so cycles in the transition graph are important to net behavior. The two kinds of cycles will be termed net cycles and state cycles, respectively. The relation between locally balanced cycles and the resulting state cycles will be a key to the behavior of more general net cycles.



d_0	d_1	$\cdots$	d_{n-2}	d_{n-1}	q_0
0	0	$\cdots$	0	0	ϵ_0
0	0	$\cdots$	0	1	ϵ_1
0	0	$\cdots$	1	0	ϵ_2
0	0	$\cdots$	1	1	ϵ_3
$\cdots$		$\cdots$		$\cdots$	$\cdots$
1	1	$\cdots$	1	0	ϵ_{2^n-2}
1	1	$\cdots$	1	1	ϵ_{2^n-1}

$\epsilon_{2j} = \bar{\epsilon}_{2j+1}$

FIG. 5. A locally balanced cycle and the truth table of its switch.

Locally balanced cycles can be defined in the following way: In the truth table corresponding to a switching element let rows $2j$ and $2j + 1$, for $j = 0, 1, \cdots, 2^{n-1} - 2$, be called simply the j^{th} *pair*. Let the function values determined by the two arguments of the j^{th} pair be ϵ_{2j} and ϵ_{2j+1}, respectively. A switching element will be called *locally balanced* if $\epsilon_{2j} = \bar{\epsilon}_{2j+1}$ for all pairs, $j = 0, 1, \cdots, 2^{n-1} - 2$. A *locally balanced cycle* satisfies the following conditions:

1. One switching element occurs in the cycle and that element is locally balanced.

2. There are $n > 0$ delay elements in the cycle. The output of the switch is identified with the input of delay d_0. The output of delay d_j is identified with the input of delay d_{j+1}, $j = 0, \cdots, n-2$.

3. The switch has n inputs. The j^{th} input of the switch (the j^{th} column of the associated truth table) is identified with the output of delay d_j, $j = 0, \cdots, n-1$.

Theorem 4. The state-transition graph of any locally balanced cycle consists only of disjoint cycles of states.

Proof:

1. Consider, at any given time t, the ordered n-tuple $(p_0(t), \cdots, p_{n-1}(t))$ of the states of the n inputs to the switch in a locally balanced cycle. By the definition of a locally balanced cycle, this n-tuple is identified with the ordered n-tuple of delay output states at time t, $(d_0(t), \cdots, d_{n-1}(t))$. Thus each of the 2^n net states of the cycle is represented by the argument part of a line of the switching element truth table.

2. If the net state of the cycle at t is given by the j^{th} line of the truth table, then the net state of the cycle at $t+1$ is simply given by the ordered n-tuple with ϵ_j as its first digit and the value of $p_i(t)$ as its $(i+1)^{th}$ digit. That is,

$$(d_0(t+1), \cdots, d_{n-1}(t+1)) = (\epsilon_j, p_0(t), \cdots, p_{n-2}(t))$$

where the argument of line j has in effect been "shifted one to the right," ϵ_j being "shifted in," $p_{n-1}(t)$ being "shifted out." The truth table, thus extended, becomes the *derived transition table* for the locally balanced cycle (Table II).

TABLE II.—*The Derived Transition Table of a Locally Balanced Cycle.*

$s(t)$*				$s(t+1)$			
$d_0(t) = p_0(t)$	$\cdots$	$d_{n-2}(t) = p_{n-2}(t)$	$d_{n-1}(t) = p_{n-1}(t)$	$d_0(t+1) = q(t)$	$d_1(t+1) = p_0(t)$	$\cdots$	$d_{n-1}(t+1) = p_{n-2}(t)$
0	$\cdots$	0	0	ϵ_0	0	$\cdots$	0
0	$\cdots$	0	1	ϵ_1	0	$\cdots$	0
0	$\cdots$	1	0	ϵ_2	0	$\cdots$	1
0	$\cdots$	1	1	ϵ_3	0	$\cdots$	1
	$\cdots$					$\cdots$	
1	$\cdots$	1	0	ϵ_{2^n-2}	1	$\cdots$	1
1	$\cdots$	1	1	ϵ_{2^n-1}	1	$\cdots$	1

* $s(t)$ is the net state of the cycle at time t.

3. Now, the j^{th} pair of the locally balanced switch gives rise to a pair of successor n-tuples, $s_{2j}(t+1) = (\epsilon_{2j}, p_0(t), \cdots, p_{n-2}(t))$ and $s_{2j+1}(t+1) = (\epsilon_{2j+1}, p_0(t), \cdots, p_{n-2}(t))$. s_{2j} and s_{2j+1} have identical digits in the last $n-1$ places by step (2); furthermore no other pair of successors has the same ordered set of digits in the last $n-1$ places. The first digit of s_{2j} is the binary complement of the first digit of s_{2j+1} since $\epsilon_{2j} = \bar{\epsilon}_{2j+1}$ by definition of a locally balanced switch. Therefore the argument states of the j^{th} pair map into distinct n-tuples, s_{2j} and s_{2j+1}, which occur nowhere else on the right of the derived transition table (Table II). In other words, the derived transition table is a $1-1$ mapping of the 2^n net states onto themselves. Such a mapping is a permutation on the net states and, by elementary group theory, permutations can always be reduced to a product of disjoint permutation cycles. These permutation cycles correspond directly to disjoint state cycles of the transition graph.

Note that the state-transition graph of an input-free cycle consists only of disjoint state cycles just in case the cycle is backwards deterministic in the sense of Burks and Wang (1, p. 286). Using this fact, and noting that $\epsilon_{2j} = \epsilon_{2j+1}$ implies $s_{2j} = s_{2j+1}$, we can restate Theorem 4 in a stronger form:

Theorem 4'. Let C be an n-delay cycle with one (arbitrarily chosen) n-input switch which is connected just like the switch in a locally balanced cycle. C will be backwards deterministic if and only if the switch is locally balanced.

In what follows, a pair will be said to be *normally oriented* if $\epsilon_{2j} = 0$, $\epsilon_{2j+1} = 1$. A pair will be said to be *inversely oriented* if $\epsilon_{2j} = 1, \epsilon_{2j+1} = 0$. The simplest locally balanced cycles result when the pairs associated with the switch are either all normally oriented or all inversely oriented. When this is the case the output of the switch, $q(t)$, is independent of all argument columns except the last, $p_{n-1}(t)$. Thus in effect the cycle is an input-free simple cycle. The next theorem gives some properties of the state-transition graphs of these simplest locally balanced cycles.

Theorem 5. Let L be a locally balanced cycle with n delays. If all the pairs of the switch are normally oriented, a state cycle with exactly p states occurs if and only if $p = 1$ or, for $p > 1$, g.c.d. $(p, n) = p$. There will be two state cycles with $p = 1$ and, for $p > 1$, there will be $n_p = \dfrac{2^p - 2^{p'}}{p}$ state cycles, where p' is the next number smaller than p which is the length of a state cycle. If all the pairs of the switch are inversely oriented, a state cycle with exactly p states occurs if and only if g.c.d. $(p, 2n) = p$ and p does not divide n. The number of state cycles having p elements is again given by n_p.

The state cycles of the locally balanced cycle with a normally oriented switch, which I will call *normal state cycles*, figure basically in the present study. Part of the reason for this lies in the following operation: an *inversion* consists in changing a given pair of a locally balanced switch from normally oriented to inversely oriented or *vice versa*. The result of an inversion is a new locally balanced switch produced from the given one. It follows directly from the definition of a locally balanced switch that any locally balanced switch can be transformed into any other by a succession of inversions. Thus, for example, any locally balanced switch can be produced by using a succession of inversions on a normally oriented switch. The next five theorems will explore the relations between normal state cycles, inversions, and the transition graphs of locally balanced cycles.

In the proofs, and at other points from here on, the state represented by a given binary n-tuple will, where convenient, be labelled by the decimal equivalent of the corresponding binary number. Thus $(0, 0, \cdots, 0)$ becomes 0, $(0, \cdots, 0, 1, 0)$ becomes 2, and $(1, 1, \cdots, 1)$ becomes $2^n - 1$.

Theorem 6. Let E_1 be the switching element of a locally balanced cycle; let E_2 be the switching element derived from E_1 by an inversion on the j^{th} pair, that is, on $(\epsilon_{2j}, \epsilon_{2j+1})$; and let s_{2j} and s_{2j+1} be the arguments of the j^{th} pair. If s_{2j} and s_{2j+1} belong to different state cycles, C_1 and C_2, in the transition graph with respect to E_1, then the transition graph with respect to E_2 will be the same as that for E_1 *except* that C_1 and C_2 will be united into a single state cycle consisting exactly of all of the states belonging to C_1 and C_2. If s_{2j} and s_{2j+1} belong to the same state cycle, C, in the transition graph with respect to E_1, then the transition graph with respect to E_2 will be the same as that for E_1 *except* that C will be separated into two disjoint state cycles which together include all of the states belonging to C.

Proof:

The transition table for a locally balanced cycle, as derived from the switching element's truth table, is unchanged by an inversion except for the lines corresponding to the inverted pair. Let s_{2j} and s_{2j+1} be the left-hand entries of these two lines and s'_{2j}, s'_{2j+1} their respective successors (right-hand entries) before the inversion. After the inversion the successor of s_{2j} will be s'_{2j+1} and the successor of s_{2j+1} will be s'_{2j}. From Theorem 4 one of two cases must hold for s_{2j} and s_{2j+1}, either they belong to different state cycles or else they belong to the same state cycle.

Case 1. s_{2j}, s_{2j+1} belong to different state cycles C_1, C_2.

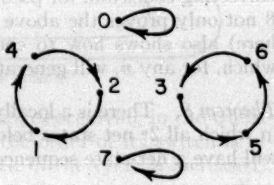
After the inversion the succession from s'_{2j} to s_{2j} within C_1 is unchanged, thus each element of C_1 appears in turn (since there were no elements of C_1 between s_{2j} and s'_{2j} all are present in this succession). However, the successor of s_{2j} is s'_{2j+1} which belongs to C_2. The succession from s'_{2j+1} to s_{2j+1} is undisturbed and every element of C_2 appears in this succession. Finally the successor of s_{2j+1} is s'_{2j} which completes the new cycle (since we began the succession with s'_{2j}). All elements of C_1 and C_2 belong to the resulting state cycle.

Inversions

Derived Transition Table

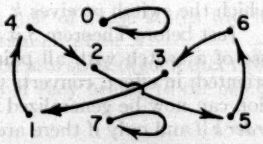
	d_0	d_1	d_2	q_0	d_1	d_2
None						
Pair 0	0	0	0	$e_0 = 0$	0	0
	0	0	1	$e_1 = 1$	0	0
Pair 1	0	1	0	$e_2 = 0$	0	1
	0	1	1	$e_3 = 1$	0	1
Pair 2	1	0	0	$e_4 = 0$	1	0
	1	0	1	$e_5 = 1$	1	0
Pair 3	1	1	0	$e_6 = 0$	1	1
	1	1	1	$e_7 = 1$	1	1

Transition Graph



Pair 1 inverted

After inversion of pair 1 derived transition table is the same except $e_2 = 1$ and $e_3 = 0$.



Pairs 1 and 2 inverted

After inversion of pairs 1 and 2 transition table is the same except $e_2 = 1$, $e_3 = 0$, $e_4 = 1$, and $e_5 = 0$.

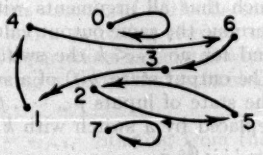


FIG. 6. Effects of inversions on the transition graph of a locally balanced cycle.

Case 2. s_{2j}, s_{2j+1} belong to the same state cycle C.

Let the segment of C from s_{2j} through s_{2j+1} be D_1 and the other segment from s_{2j+1} through s_{2j} be D_2. The first element of D_1 is s'_{2j} and the last s_{2j+1}. After inversion the succession in D_1 is unchanged but s'_{2j} becomes the successor of s_{2j+1}. Thus D_1 becomes a state cycle. Similarly D_2 becomes another, disjoint, state cycle. The effect of the inversion has been to "pinch" the original cycle in two at s_{2j}, s_{2j+1}.

Theorem 7. In any transition graph containing normal cycles, there are at most two inversions which can, individually, connect a given pair of normal cycles (C_1, C_2). There are no inversions which can separate a given normal cycle into two state cycles.

The next theorem shows the application of the inversion operation to the solution of a combinatorial problem earlier investigated and solved by N. G. de Bruijn (3). The problem is to show, for any n, that there is an ordered cycle of 2^n digits 0 or 1 such that the 2^n possible ordered sets of n consecutive digits of that cycle are all different. Logical nets which can generate such sequences are of interest for error-correcting codes and for pseudo-random number generation. Theorem 8 not only proves the above statement, but in its proof (not included here) also shows how to construct input-free locally balanced cycles which, for any n, will generate such sequences as output.

Theorem 8. There is a locally balanced cycle having a transition graph in which all 2^n net states belong to a single state cycle; that is, the net will have a net-state sequence of period 2^n.

The next theorem begins a direct investigation of the effect of increasing the number of feedback loops in a cycle. The theorem basically concerns input-free locally balanced cycles in which some of the feedback loops to the switch have been omitted, that is, cycles in which the switch receives $k \leqslant n$ inputs from the cycle.

Just before Theorem 5 it was noted that, in a locally balanced cycle, use of a switch with all pairs normally oriented or all pairs inversely oriented, in effect, converts the cycle to a simple cycle. This observation can now be generalized: A switching element will be said to be of *order k* if and only if there are k numbers, $0 \leqslant i_0 < \cdots < i_{k-1} \leqslant n-1$, such that all arguments with the same values for $p_{i_0}, \cdots, p_{i_{k-1}}$ determine the same output value, $q(t)$ for the switch. If this is true for k and for no $k_1 < k$ the switch will be said to be *properly of order k*. The output state, $q(t)$ of a switch properly of order k depends only on the state of inputs $p_{i_0}, \cdots, p_{i_{k-1}}$; thus in a cycle the switch could be replaced by a switch with $k \leqslant n$ inputs identified with delay outputs $d_{i_0}, \cdots, d_{i_{k-1}}$.

For a locally balanced switch, $i_{k-1} = n-1$ in the above definition. This is the case because, in each pair determined by giving values to $p_0, \cdots, p_{n-2}$, $q(t) = \epsilon_{2j}$ when $p_{n-1} = 0$ and $q(t) = \epsilon_{2j+1} = \bar{\epsilon}_{2j}$ when $p_{n-1} = 1$. That is, different values of p_{n-1} give rise to different values of $q(t)$. Using these facts, the definition of order can be recast for locally balanced switches in terms of orientation of pairs: A locally balanced switch is of order k if and only if there are $k-1$ numbers, $0 \leqslant i_0 < \cdots < i_{k-2} < n-1$, such that all pairs with the same values for $p_{i_0}, \cdots, p_{i_{k-2}}$ have the same orientation.

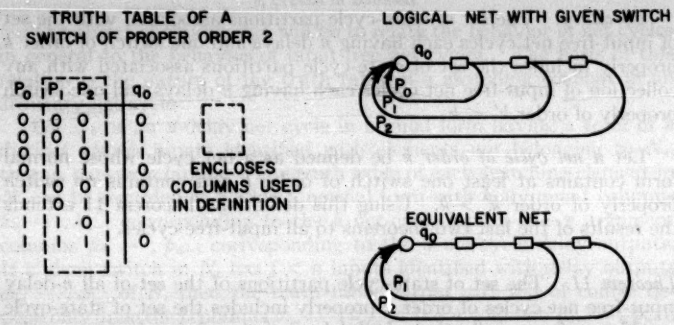


FIG. 7. A switch properly of order 2.

Theorem 9 will describe some of the changes occurring in the transition graph of a locally balanced cycle as the order of its switch is increased (that is, as the amount of feedback in the cycle is increased). The statement of this theorem as well as that of some succeeding theorems can be considerably shortened by making the following definition: The *state-cycle partition* of a net N with n delays is a partition of the set of 2^n net states satisfying the following conditions:

1. One subset of the partition consists just of those states which do not belong to a state cycle in the transition graph of N.
2. The remaining net states are separated into subsets such that two states belong to the same subset if and only if they belong to the same state cycle in the transition graph of N.

Theorem 9. The set of state-cycle partitions associated with the set of locally balanced cycles having n delays and a switching element of order k properly includes the set of state-cycle partitions associated with any collection of locally balanced cycles having n delays and switching elements properly of order $k' < k$.

Let a pair be called *unbalanced* if $\epsilon_{2j} = \epsilon_{2j+1}$. Starting from the normally oriented n-input switch any n-input switch can be produced by unbalancing selected pairs after carrying out a properly chosen set of inversions. Using this fact, Theorem 10 and its corollary extend the results of Theorem 9 to every input-free cycle with one switch.

Theorem 10. The set of state cycles for a given input-free cycle with one switch, E_1, of order k is a subset of the set of state cycles of an associated locally balanced cycle with a switch, E_2, or order $k' \geq k$. No locally balanced cycle with a switch of order less than k' includes, as a subset, the set of state cycles of the cycle with switch E_1.

Corollary. The set of state-cycle partitions associated with the set of input-free net cycles each having n delays and one switch of order k properly includes the set of state-cycle partitions associated with any collection of input-free net cycles each having n delays and one switch properly of order $k' < k$.

Let a *net cycle of order* k be defined as a net cycle whose normal form contains at least one switch of order k and contains no switch properly of order $k' > k$. Using this definition, Theorem 11 extends the results of the last two theorems to all input-free cycles.

Theorem 11. The set of state-cycle partitions of the set of all n-delay input-free net cycles of order k properly includes the set of state-cycle partitions of any collection of n'-delay, $n' \leqslant n$, input-free net cycles of order $k' < k$.

Intuitively, Theorem 11 says that for input-free cycles, there is no upper limit of complexity c such that the behavior of any given input-free cycle can be realized by some cycle of complexity $c' < c$. Here complexity is defined as a number pair (n, k), where n is the number of delays in the cycle, and k is the maximum of the numbers k_j, $j = 0, 1, \cdots, n-1$, where k_j is the number of delays in the cycle which feed back their outputs to delay j.

We can look at this result in another way:

Consider the case of an experimenter presented with a black box (cf. Moore's gedanken experiments (6)) about which he is given the following information:

1. all elements in the box belong to a single input-free net cycle,
2. the box has one output for each delay element inside,
3. at any time the box can be set to an arbitrary "initial" net state and observed for as long as desired.

Theorem 11 tells us that there is a net with at most k feedback loops through each switch which will make the black box behave in a fashion impossible for any net with $n' \leq n$ delays and $k' < k$ feedback loops through each switch. That is, for each level of complexity $c = (n, k)$ there are input-free cycles of complexity c which can be distinguished by the experiment from any of complexity $c' < c$. Moreover, the set of behaviors possible for black boxes of complexity (n, k) properly includes the set of behaviors possible for boxes having $n' \leq n$ delays and $k' < k$ feedbacks to each switch.

Theorem 12 in the next section makes direct use of Theorem 11 to prove the same statements for cycles with input.

5. CYCLES IN GENERAL

The object of this section will be to relate the behavior of net cycles in general to the behavior of input-free net cycles. The basis of this relation is the nature of the truth table of a switching element in an arbitrary net cycle:

Let N_c be an n-delay net cycle in normal form having a total of k distinct switch inputs identified with elements not belonging to N_c, that is, net-cycle inputs. The truth table of each switching element in N_c can be regarded as having a normal form with k argument columns $h_0, \cdots, h_{k-1}$ corresponding to the k net-cycle inputs and n argument columns $p_0, \cdots, p_{n-1}$ corresponding to the n net-cycle delay outputs. If a given switch in N_c has $j < n$ inputs identified with delay outputs $d_{i_0}, \cdots, d_{i_{j-1}}$ of N_c then the truth table output $q(t)$ will of course depend only upon the j columns $p_{i_0}, \cdots, p_{i_{j-1}}$ of the n columns $p_0, \cdots, p_{n-1}$. That is, with respect to the n columns $p_0, \cdots, p_{i_{j-1}}$, the normal form of the truth table will be of order j. Similarly, if the given switch has $b < k$ net-cycle inputs then $q(t)$ will depend only upon b columns $h_{r_0}, \cdots, h_{r_{b-1}}$ of the k columns $h_0, \cdots, h_{k-1}$. Each switch in the net cycle, regardless of the number of its inputs, can thus be given a standard truth table with $k + n$ argument columns and one output column.

Note that the $k + n$ argument columns of the truth table of each switch in N_c are identical when they are given the order $h_0, \cdots, h_{k-1}$, $p_0, \cdots, p_{n-1}$. In the normal form of the net cycle N_c, each delay in the cycle, d_i, has its input identified with the output of one of the switches, q_i, in the cycle so that $d_i(t + 1) = q_i(t)$. Furthermore, $p_i(t) = d_i(t)$. If the n switch output columns are arranged in the order $q_{n-1}, q_0, q_1, \cdots, q_{n-2}$ at the right of the $k + n$ argument columns the result

TABLE III.—*Derived Transition Table for a General Cycle with Input.*

$I(t)$			$s(t)$			$s(t+1)$		
$h_0(t)$	$\cdots$	$h_{k-1}(t)$	$d_0(t) = p_0(t)$	$\cdots$	$d_{n-1}(t) = p_{n-1}(t)$	$d_0(t+1) = q_{n-1}(t)$	$\cdots$	$d_{n-1}(t+1) = q_{n-2}(t)$
0	$\cdots$	0	0	$\cdots$	0	$\epsilon_{n-1,0}$	$\cdots$	$\epsilon_{n-2,0}$
0	$\cdots$	0	0	$\cdots$	1	$\epsilon_{n-1,1}$	$\cdots$	$\epsilon_{n-2,1}$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
0	$\cdots$	0	1	$\cdots$	1	$\epsilon_{n-1,2^n-1}$	$\cdots$	$\epsilon_{n-2,2^n-1}$
0	$\cdots$	1	0	$\cdots$	0	$\epsilon_{n-1,2^n}$	$\cdots$	$\epsilon_{n-2,2^n}$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
1	$\cdots$	1	0	$\cdots$	0	$\epsilon_{n-1,2^{k+n-1}}$	$\cdots$	$\epsilon_{n-2,2^{k+n-1}}$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
1	$\cdots$	1	1	$\cdots$	1	$\epsilon_{n-1,2^{k+n}-1}$	$\cdots$	$\epsilon_{n-2,2^{k+n}-1}$

is a transition table where the transition from $(d_0(t), \cdots, d_{n-1}(t))$ to $(d_0(t+1), \cdots, d_{n-1}(t+1))$ depends upon the value of the net-cycle input state $(h_0(t), \cdots, h_{k-1}(t))$.

Now fix a value, I_0, for $I(t) = (h_0(t), \cdots, h_{k-1}(t))$ and consider, in the derived transition table for N_c, the 2^n rows having the given values for the arguments $h_0(t), \cdots, h_{k-1}(t)$. The 2^n rows so selected constitute the transition table of an n-delay net cycle N_{I_0} which has n switches in normal form, each with n inputs. Of necessity, N_{I_0} is an input-free cycle in which the switch at position q_i has, for arguments $p_0(t), \cdots, p_{n-1}(t)$, an output $q_i(t) = \epsilon_{i,x \cdot 2^n+y}$, where x = the decimal equivalent of the binary number $h_0(t) \cdot 2^{k-1} + \cdots + h_{k-1}(t) \cdot 2^0$ and y = the decimal equivalent of $p_0(t) \cdot 2^{n-1} + \cdots + p_{n-1}(t) \cdot 2^0$. That is, at each position in N_{I_0} we have a switch whose output for an argument $(p_0(t), \cdots, p_{n-1}(t))$ is simply the value $q_i(t)$ given for the corresponding n argument values of $p_0, \cdots, p_n$ in one of the selected 2^n rows. We see that the effect on N_c of a given net-cycle input state I_0 is to select an n-input switch at each position in N_c, the result being an n-delay input-free cycle N_{I_0}. If $I(t) = I_0$ then the behavior of the cycle N_c for that one moment of time will be exactly that of N_{I_0}.

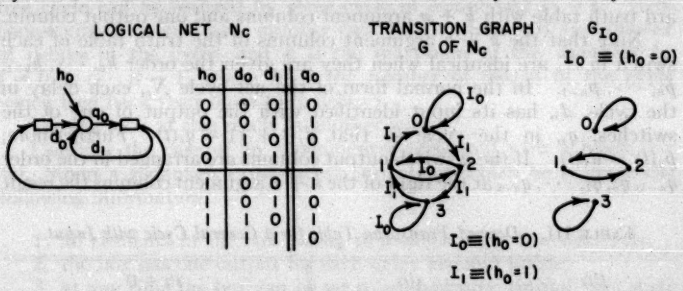


FIG. 8. An example of the selection of $G_{I(t)}$ by the input-state $I(t)$.

The importance of the preceding observation lies in the relation between the transition graphs of the various $N_{I(t)}$ and the transition graph of the cycle N_c. Let G be the transition graph of N_c and let the 2^k possible net-cycle input states of N_c, $(h_0, \cdots, h_{k-1})$, be labelled $I_0, I_1, \cdots, I_{2^k-1}$. Let G_{I_j} be the subgraph of G obtained by retaining all of the vertices of G and only the edges of G labelled I_j (see Section 2). Then G_{I_j} is exactly the transition graph of the net cycle N_{I_j}, the input-free net cycle selected when $I(t) = I_j$. Conversely, if the 2^k graphs G_{I_j}, $j = 0, \cdots, 2^k - 1$ are given, then the graph G can be constructed. This is done by simply superposing all the graphs G_{I_j} so that vertices with the same label are identified and each edge for each G_{I_j} appears in the result, G, connecting the same vertices. In the process of forming

G from the G_{I_i} many of the properties common to all the G_{I_j} will reappear as properties of G.

In the special case that a subgraph G_{I_j} of the transition graph G consists just of state cycles, it can be conveniently represented by an element of the group of permutations on 2^n elements. To do this we make use of the fact that any permutation can be given by the product of a set of disjoint circular permutations: Let C_i be the i^{th} state cycle of G_{I_j} (under some arbitrary ordering). Let $s_{i,0}, s_{i,1}, \cdots, s_{i,n_i}$ be the net states belonging to C_i ordered so that $s_{i,y+1}$ succeeds $s_{i,y}$ (and $s_{i,0}$ succeeds s_{i,n_i}) in G_{I_j}. The circular permutation $(s_{i,0}, s_{i,1}, \cdots, s_{i,n_i})$ represents the state cycle C_i and, thus, the group element

$$g_j = (s_{0,0}, s_{0,1}, \cdots, s_{0,n_0})(s_{1,0}, \cdots, s_{1,n_1}) \cdots (s_{v,0}, \cdots, s_{v,n_v})$$

represents G_{I_j}. Note that, if the state cycles C_i are normal state cycles, the operation of inversion on the pair of states s_{2h} and s_{2h+1} can be represented by multiplying g_j on the left by the transposition (s_{2h}, s_{2h+1}). Thus, if the switch in a locally balanced cycle is obtained by inversions on pairs $h_0, h_1, \cdots, h_r$ of a normally oriented switch, the resulting transition graph G_{I_j} will be represented by the group element

$$g'_j = (s_{2h_0}s_{2h_0+1})(s_{2h_1}s_{2h_1+1}) \cdots (s_{2h_r}s_{2h_r+1}) \cdot g_j$$

where g_j represents a transition graph consisting just of normal state cycles.

If G is the transition graph of a net cycle N_c with input and if each subgraph G_{I_j} of G consists just of state cycles, then given an input-state sequence of period m the resulting net-state period of N_c can be determined by means of the group representation. This is accomplished by using the graphs $G_{I(0)}, G_{I(1)}, \cdots, G_{I(m-1)}$ specified by input states $I(0), I(1), \cdots, I(m-1)$, where $I(t) = I(j)$ if and only if $t \equiv j \bmod m$, $j = 0, \cdots, m-1$. If $g_{I(i)}$ is the group element corresponding to $G_{I(i)}$, then the net-state period will be a divisor of the product $m \cdot r_g$, where r_g is the order of the group element

$$g = g_{I(0)} \cdot g_{I(1)} \cdots g_{I(m-1)}.$$

As a more general example of the way properties of the G_{I_j} reappear in G, Theorem 11 will be applied to net cycles in general. First, the definition of a cycle of order r (given in Section 4) must be extended appropriately: Let E_1 be an arbitrary switching element with $b = n+k$ inputs. Let n of these inputs, $p_0, \cdots, p_{n-1}$ be identified with the outputs of elements belonging to a cycle N_c. Let k of the inputs $h_0, \cdots, h_{k-1}$ be identified with the outputs of elements not belonging to N_c. E_1 will be said to be of *order r w.r.t. a cycle* N_c if, ignoring the argument columns

$h_0, \cdots, h_{k-1}$, it is of order r when just columns $p_0, \cdots, p_{n-1}$ are considered. A net cycle N_c, with or without net-cycle inputs, will be an (n, r)-*cycle* just in case the normal form of the cycle contains exactly n delays, at least one switch of order r w.r.t. the cycle N_c, and no switches of proper order $r' > r$ w.r.t. the cycle. An (n, r)-cycle will, in effect, have no switch which receives more than r distinct feedbacks from delays in the cycle.

Theorem 12. The set of state-cycle partitions of the set of all (n, r)-cycles properly includes the set of state-cycle partitions of any collection of (n', r')-cycles with $n' \leqslant n$, $r' < r$.

Proof:

1. In order to apply Theorem 11 to an arbitrary net cycle, N_c, of order r, consider first the transition graph G_{I_j} of N_c. Each G_{I_j} will be the transition graph of an input-free cycle N_{I_j} of order r (since the switches of N_{I_j} cannot have any cycle feedbacks not present in N_c). Thus each G_{I_j} will be subject to Theorem 11 as applied to input-free net cycles of order r. The result will be that each G_{I_j} can satisfy the following three conditions: (1.1) G_{I_j} consists just of disjoint state cycles; (1.2) All normal state cycles with more than $n - r + 1$ ones are present in G_{I_j} (all states of a given normal cycle have the same number of ones in their coded form; when this number is i the normal state cycle will be said to contain i ones—see proof of Theorem 9 for exact discussion); and (1.3) One normal state cycle with $n - r + 1$ ones is not present in G_{I_j}.

2. It follows from step (1) and the discussion preceding this theorem that there is a cycle N_c of order r with the following properties for its transition graph G: (2.1) All subgraphs G_{I_j} of G consist only of disjoint state cycles; (2.2) For each state s with $i > n - r + 1$ digits equal to 1 there is a unique state s' with i digits equal to 1 which succeeds s no matter what the input state $I(t)$ is. The cycle of states so determined has n states, or a divisor thereof, as elements; (2.3) There is a state s_0 with $i_0 = n - r + 1$ digits equal to 1 which, for some input state I_0, has a successor state s'_0 with $i_0 - 1$ digits equal to 1. The state cycle to which s_0 belongs has $n_0 > n$ elements.

3. Using the state-cycle partition (see the definition preceding Theorem 9—noting that a state cycle in the transition graph is defined analogously to a net cycle in a logical net) properties (2.1)–(2.3) can be restated: (3.1) The state-cycle partition of G contains no subset of elements not belonging to a state cycle; (3.2) Each net state with more than $n - r + 1$ digits equal to 1 belongs to a subset of the partition which contains exactly n states or a divisor thereof; (3.3) Some net state with $n - r + 1$ digits equal to 1 belongs to a subset with more than n states.

4. For an n-delay cycle N'_e of order $r' < r$ no derived transition graph G'_{Ij} can satisfy all three conditions (1.1)–(1.3) that each G_{Ij} satisfies (by Theorem 11). Hence no n-delay net cycle of order $r' < r$ can have a transition graph satisfying the conditions (2.1)–(2.3). Thus, in turn, no n-delay net-cycle of order $r' < r$ can exhibit the state-cycle partition of step (3).

5. The remainder of the proof follows the argument of step (12) of Theorem 11.

Theorem 12 is the counterpart of Theorem 11 for arbitrary cycles with input. Intuitively it says that there is no level of complexity c such that any behavior possible for a cycle can be realized by a cycle of complexity $c' < c$ (cf. the second question asked at the beginning of Section 1).

Theorem 12 can be interpreted in Moore's framework in much the same way Theorem 11 was. Let $B_j(n, k)$ be a black box having the following properties: (1) n observable outputs each of which is a delay element output; (2) k inputs (net inputs) whose states at $t = 0, 1, 2, \cdots$ are specified by the observer; (3) an arbitrary number of elements in the box all belonging to one and the same (n, r)-cycle.

The set of behaviors possible for the set of all $B_j(n, k)$ which contain an (n_0, r_0)-cycle properly includes the set of behaviors possible for any collection of $B_j(n, k)$ which contain an (n_1, r_1)-cycle such that $n_1 \leqslant n_0$ and $r_1 < r_0$. In other words, no cycle with at most n delays or k inputs and less than r feedbacks to each switch can imitate the behavior of particular cycles with n delays, k inputs and r feedbacks to one or more switches.

The results of Sections 4 and 5 lend strong support to the following stronger conjecture:

For any (n, r) there is some transformation not realized by any net containing only (n, r)-cycles.

In fact it should be possible to construct a lattice of behaviorial transformations defined as follows: Let $N_{(n,r)}$ be the set of all logical nets containing only (n, r) cycles. With each logical net in $N_{(n,r)}$ will be associated a transformation which gives the net-state sequence produced by each input-state sequence. Let $B_{(n,r)}$ be the set of transformations associated with the set $N_{(n,r)}$. The lattice should satisfy the following conditions: (1) $B_{(n,r)}$ properly includes $B_{(n',r')}$ if $n' < n$ and $r' \leqslant r$ or if $n' \leqslant n$ and $r' < r$; (2) g.l.b. $[B_{(n_1,r_1)}, B_{(n_2,r_2)}] = B_{(n_0,r_0)}$ where $n_0 = \min(n_1, n_2)$ and $r_0 = \min(r_1, r_2)$; (3) l.u.b. $[B_{(n_1,r_1)}, B_{(n_2,r_2)}] = B_{(n_3,r_3)}$ where $n_3 = \max(n_1, n_2)$ and $r_3 = \max(r_1, r_2)$.

It seems that the interrelations between periodic input and net-state sequences, net cycles, state cycles, and permutation cycles, as sketched in this paper, could provide a basis for the proof of this conjecture.

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BARTOL RESEARCH FOUNDATION
MARTIN A. POMERANTZ, DIRECTOR

SOLAR-PRODUCED COSMIC RADIATION NEAR THE GEOMAGNETIC POLE ON MAY 4, 1960*

BY

MARTIN A. POMERANTZ AND VASANT R. POTNIS

In over twenty-five years of continuously recording cosmic radiation, intensity increases associated with solar flares and attributed to the arrival of particles from the sun had been observed at a number of stations at the surface of the earth on six occasions.

Although the sixth event (1)[1] which occurred on July 17, 1959 during a period of very unusual geomagnetic activity, differed somewhat from the previous flare-associated intensity increases in detailed structure, evidence from ionospheric-scatter signal intensity observations at 38 Mc/sec indicated that, as suggested by the neutron monitor records, the event represented the arrival from the sun of cosmic ray particles with energies above the geomagnetic cutoff (2). The atypical time behavior has been a topic of considerable discussion, but this interpretation has been supported by subsequent analysis (3).

The preceding event on February 23, 1956, was the largest ever observed. The intensity recorded by neutron monitors at some locations increased by as much as a factor of forty-six in this case. Furthermore, this is the only example of an increase which was detectable near the equator.

This note constitutes a preliminary description of the seventh event of this type which has occurred recently. On May 4, 1960, the largest sudden increase recorded in the immediate vicinity of the geomagnetic pole was reported by the Bartol Neutron Monitor Station at Thule, Greenland (geomagnetic latitude 88°N).

The relevant data are given in Table I, and plotted in Fig. 1. Barometric corrections which would introduce a negligible effect in this instance have not been applied. The average counting rate between 0900 UT and 1030 UT represented as a dash-dot line is the 100 per cent base level. The intensity reached 233 per cent at the peak. The uncertainty in the timing is less than 1 minute, although the precision in determining the time of maximum is somewhat reduced by statistical considerations.

The increase started at 1030 UT and reached its maximum at 1042 UT. The relatively broad peak persisted until the start of the decline at 1048 UT. The counting rate returned to the pre-increase level approximately three hours later.

* This work was supported in part by the National Science Foundation and the Office of Naval Research.

[1] The boldface numbers in parentheses refer to the references appended to this paper.

TABLE I.—*Thule Neutron Monitor: Cosmic Ray Increase of May 4, 1960.*

Time UT	Counts ÷ 64	Pressure, mm of Hg	Hourly Counting Rate
0900–0915	78	730.6	312
0915–0930	78		312
0930–0945	77		308
0945–1000	75		300
1000–1010	51	730.2	306
1010–1020	52		312
1020–1030	52		312
1030–1032	12		360
1032–1034	12		360
1034–1036	15		450
1036–1038	18		540
1038–1040	22		660
1040–1042	22		660
1042–1044	24		720
1044–1046	24		720
1046–1048	24		720
1048–1050	18		540
1050–1052	21		630
1052–1054	18		540
1054–1056	16		480
1056–1058	16		480
1058–1100	15		450
1100–1105	41	730.1	492
1105–1110	36		432
1110–1115	35		420
1115–1120	34		408
1120–1125	30		360
1125–1130	30		360
1130–1145	88		352
1145–1200	86		344
1200–1215	83	730.0	332
1215–1230	84		326
1230–1245	82		328
1245–1300	82		328
1300–1315	81	729.9	324
1315–1330	78		312
1330–1345	82		328
1345–1400	75		300

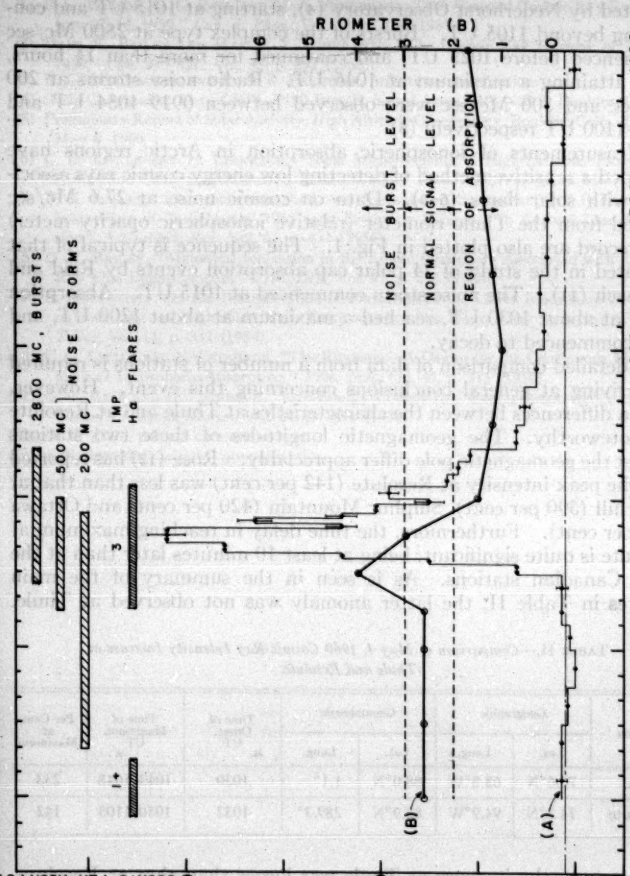

FIG. 1. Intensity of nucleonic component at Thule, Greenland, and other related phenomena on May 4, 1960.

A solar flare of Importance 3 on the west limb of the solar disc was reported by Nederhorst Observatory (4), starting at 1015 UT and continuing beyond 1105 UT. Bursts of the complex type at 2800 Mc/sec commenced before 1025 UT, and continued for more than 1½ hours, after attaining a maximum at 1046 UT. Radio noise storms at 200 Mc/sec and 500 Mc/sec were observed between 0919–1054 UT and 1015–1100 UT respectively (5).

Measurements of ionospheric absorption in Arctic regions have provided a sensitive method of detecting low energy cosmic rays associated with solar flares (6–9). Data on cosmic noise at 27.6 Mc/sec derived from the Thule riometer (relative ionospheric opacity meter) (10) record are also plotted in Fig. 1. The sequence is typical of that indicated in the study of 24 polar cap absorption events by Reid and Leinbach (11). The noise storm commenced at 1015 UT. Absorption set in at about 1030 UT, reached a maximum at about 1200 UT, and then commenced to decay.

A detailed comparison of data from a number of stations is required for arriving at general conclusions concerning this event. However, certain differences between the characteristics at Thule and at Resolute are noteworthy. The geomagnetic longitudes of these two stations nearest the geomagnetic pole differ appreciably. Rose (12) has reported that the peak intensity at Resolute (142 per cent) was less than that at Churchill (390 per cent), Sulphur Mountain (420 per cent) and Ottawa (298 per cent). Furthermore, the time delay in reaching maximum at Resolute is quite significant, being at least 10 minutes later than at the other Canadian stations. As is seen in the summary of the main features in Table II, the latter anomaly was not observed at Thule.

TABLE II.—*Comparison of May 4, 1960 Cosmic Ray Intensity Increase at Thule and Resolute.*

Station	Geographic		Geomagnetic		Time of Onset, UT	Time of Maximum, UT	Per Cent at Maximum
	Lat.	Long.	Lat.	Long.			
Thule	76.6°N	68.8°W	88.0°N	1.1°	1030	1042–1048	233
Resolute	74.7°N	94.9°W	82.9°N	289.3°	1032	1050–1105	142

Furthermore, the increase at Thule was larger than that at Resolute. Sandström (13) has reported that at Uppsala (geomagnetic latitude 58.6°N) the peak intensity was only about 111 per cent. This marked spatial dependence will doubtless be the subject of considerable analysis.

Appreciation is expressed to the U. S. Air Force ARDC for operating the station at the G.R.D. Polar Research Facility, and to John Jones for observing the event and rapidly transmitting the relevant data.

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THE FRANKLIN INSTITUTE LABORATORIES
NICOL H. SMITH, DIRECTOR

GLAUCOMA AND VIBRATION TONOMETRY

BY
C. W. HARGENS, III

Since 1956 a group at The Franklin Institute has carried on research intended to find a superior method of glaucoma detection. This interest originated with the conception of a new principle of tonometry based upon microscopic mechanical vibrations of the eyeball. Early experiments were conducted by K. Sittel at the Institute with a driven mass and steers' eyes in which the intraocular pressure was controlled. These showed the definite relation between resonance of the eye-mass combination and the intraocular pressure. The phenomenon is logical since both mass and elasticity are present, and the latter varies with the eye's internal pressure. Since that demonstration the work has been accelerated.

One important quality of the tonometer principle involving resonance of the eyeball is that it may be possible to make measurements through the closed eyelid. This would enormously aid glaucoma screening programs by making checks of intraocular pressure rapid and safe.

The intent of the Institute group at present is to carry on research which will definitely establish the capabilities and limitations of vibration tonometry. The work is being undertaken by physical scientists and engineers of the Institute's Laboratories for Research and Development aided and advised by ophthalmologists engaged in related research at eye hospitals. At the moment, problems familiar to the physicist and engineer are being examined.

PRESENT SCOPE

Since December, 1959, an investigation of the physical principles involved in vibration tonometry has received financial support from the National Institutes of Health and the National Council to Combat Blindness. The program has a schedule of at least four years.

Investigations in the laboratory at this stage include simulation techniques. Liquid-filled, plastic enclosures (Fig. 1) are being studied in their various modes of oscillation. In all cases the pressure of the fluid, corresponding to intraocular pressure, is monitored and controlled. Thus the phenomenon of resonance as it is related to pressure and other structural details is being explored.

The use of models, usually scaled upward five to ten times eye dimensions, permits a variation in parameters not possible in experiments with animal eyes. Variation of the liquid enclosures in a quantitative way may determine whether the vibration principle will be useful in investigating fundamental properties of the eye as well as in clinical

tonometry. Moreover, it may be supposed that there are less evident parameters whose contributions to the vibrating system have pathological significance.

Of particular interest in this connection is the decrement of oscillations following an excitation. This quality, related to the "Q" of an electric analog, may be expected to represent a significant characteristic of the eye.

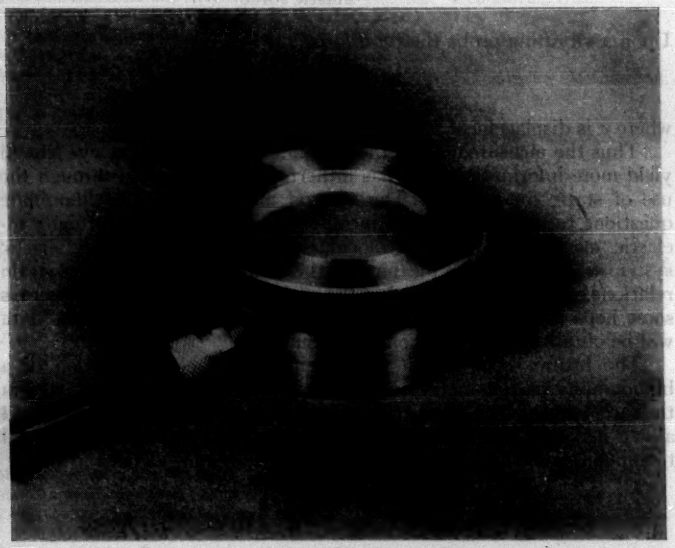

FIG. 1.

FUTURE SCOPE

A useful concept which now appears applicable is to consider the eye as a viscoelastic body. That is, the eye has both elasticity and viscosity. One may incur some criticism in doing this, because the eye is certainly not isotropic. At best it has some degree of symmetry; and that is all we can say in defense of this approach, except to promise always to consider it from the same orientation.

If one treats the eye as a viscoelastic body, one may analyze its dynamic properties. This idea has its origin in the electrical analog mentioned above, a passive circuit with reactance and losses. Here, however, we refer to the complex moduli of materials. These are: the elastic modulus, G, and the viscosity, η. As the electrical engineer finds

it useful to know how closely his circuit approaches a pure resistance or a pure inductance, it may be useful to know how a particular eye's behavior tends toward a pure spring or a Newtonian liquid.

The quantities above may be expressed in terms of a phase angle, δ, which should be measurable with suitably designed instrumentation. Here,

$$\tan \delta = \frac{\text{viscous force}}{\text{elastic force}}.$$

Upon a vibrating probe the total force,

$$F = G\sigma + \eta\dot{\sigma}$$

where σ is displacement.

Thus the measurement of the dynamic properties of the eye should yield more information than has hitherto been obtainable through the use of static tonometers. The method may also tend to illuminate questions resulting from high intraocular pressures in abnormally inelastic eyeballs and low intraocular pressures in unusually elastic systems. Although the meaning of the complex dynamic stress-strain relations cannot be told ahead of the gathering of specific data, there is some hope that new eye research techniques will evolve. These data will be obtained eventually on live tissue.

The future scope of the project includes the continuation of a bibliography on tonometry and eye tissue studies which was started in the early stages of the work. If the fundamental investigation described above makes it possible to set the specifications for an experimental tonometer, work on such an instrument will begin.

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READER OPINION

CLINICAL EFFECTS OF ARTIFICALLY PRODUCED NEGATIVE IONS ON RESPIRATORY PATIENTS *

W. Wesley Hicks [1]

A comprehensive, documented report by the Wesix Foundation to the American Medical Association (**2**)[2] on the results of investigations of air ionization from 1931 to 1958 in Germany, France, Japan, Argentina, the United States and Russia, together with very recently reported ionization studies in this country, provide evidence that negative ions are beneficial when breathed by patients with several types of respiratory disease, while positive ions tend to exert a deleterious or harmful effect.

For example, Kornblueh, Griffin, Piersol and Speicher (**3, 4**), working at the University of Pennsylvania, reported that 62 per cent of 173 hay fever and asthma patients found relief by breathing negatively ionized air.

Corrado and Beckett (**5**), reporting on a double blind study of the effect of positive ions on allergic patients, reported that 40 per cent of the patients experienced an increase in upper respiratory symptoms which did not occur when the patients were exposed to negative air ions.

Winsor and Beckett (**6**), reporting on studies at the Good Samaritan Hospital, Los Angeles, reported that the breathing capacity of the observed subjects was reduced from a pre-exposure level of 35 liters per minute to 25 liters per minute during exposure to positive ions and there was also irritation of the respiratory mucosa.

Mibashan (**7, 8**) reported applying negative ions to 300 infants, children and adults with no untoward effects; and 76 per cent of his patients with respiratory affections were benefited. He also reported that of 30 hypertensive patients treated with negative air ions, 96 per cent reacted favorably.

Vasilyev (**9**) reported that negative air ions were helpful to many hundreds of people suffering from bronchial asthma, high blood pressure, or ozena. Patients who exhibited slow healing of wounds or who had ulcers of the soft tissues were also benefited when exposed to negative ions by improvement in healing of the respective lesion.

Following his recent trip to Russia, Kornblueh (**10**) reported that six hundred medical students graduating annually from the Pavlov Institute Medical School and all resident physicians are getting a substantial indoctrination in the physical and biological aspects of ionization. In some other medical schools the same policy is being observed so that the number of students and physicians who are acquainted with this modality is steadily growing. At the

* This is presented in the form of a sequel to the author's paper published in the JOURNAL for February 1956.

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[2] The boldface numbers in parentheses refer to the references appended to this Note.

present time there are about one hundred hospitals or other health centers throughout the Soviet Union where ionization is being used.

Krueger and Smith (**11**) recently reported experimental evidence for the idea that negatively-charged oxygen is the physiological mediator in negatively-ionized air, whereas positively-charged carbon dioxide is the active component of positively-ionized air.

In further studies, Krueger, Smith, Hildebrand and Meyers (**12**) reported reduction of ciliary activity, contracture of smooth muscle, ischemia and enhanced vulnerability to trauma as a result of exposure to positively charged CO_2, and Krueger, Smith and Millar (**13**) reported that positive air ions depress ciliary activity of trachea of monkeys and man, while negative ions increase it.

Krueger and Smith (**14**) have found that positive ions in excess are harmful and that they induce discomfort and reduce the efficiency of the respiratory tract. Negative ions, on the other hand, within limits, promote comfort and health and are capable of reversing certain of the effects of positive ions.

Krueger and Smith (**15, 16**) have studied in some detail the biological mechanisms of air ion action and have advanced the hypothesis that positive air ions act by causing the release of 5-Hydroxytryptamine (Serotonin). This tissue hormone when injected into animals induces all the changes noted in experiments with positive ions. Negative air ion effects apparently depend on the demonstrated ability of negative ions to accelerate cytochrome-linked oxidative reactions, thus providing a metabolic pathway for removal of 5-Hydroxytryptamine. The hypothesis is strengthened by Krueger and Smith's observations that all the tracheal effects produced by positive or negative air ions can be duplicated by drugs that operate through the depletion or accumulation of 5-Hydroxytryptamine in tissues.

The therapeutic usefulness of negative air ions has been reported from all over the world and the results reported have been confirmed by scientific experiment. Appropriately administered, negative air ionization offers a therapeutic modality which is a simple, safe, and scientific method for affording relief to patients afflicted with certain respiratory ailments.

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[3] The source of negatively ionized air in all these references, with the exception of **7, 8,** and **9,** was the IONAIRE.

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BOOK REVIEWS

THE EDGE OF OBJECTIVITY: AN ESSAY IN THE HISTORY OF SCIENTIFIC IDEAS, by Charles Coulston Gillispie, 562 pages, diagrams, 5½ × 8½ in. Princeton, Princeton University Press, 1960. Price, $7.50.

Objectivity and rationalism have been periodically assailed by attackers from various quarters, especially those rooted in romantic and pre-romantic intuitionism. Prof. Gillispie's essay in defense of cool objectivity and its efficient sharp edge is a warm and original apology. It reasserts the conscious awareness and objectivity of some of the most brilliant and creative scientists of all times, from Galileo, Kepler, Newton, Lavoisier, Priestley to Mendel, Darwin, Faraday, Maxwell and Einstein.

The style and the author's views are captivating and frequently original. The commonplace statements about science and the scientific methods are carefully avoided. Many important scientists and some scientific ideas have not been considered at all. The author has intelligently chosen his material and presented it as an organic whole, with enthusiasm and skill. The analysis of the particular approach of a scientist and of the creative phases of his career is both profound and fascinating. For example, the mathematical innocence and extraordinary experimental skill of Michael Faraday are beautifully illustrated: "He was a Victorian Kepler, following on no Pythagoras, and knowing no geometry—a laboratory Leonardo who could see and not draw." ". . . he next tried to make the wire or magnet rotate in place in the center of the cup of mercury. He failed and—this was how his mind always worked—suspected that perhaps the current turns within the conductor. So he bent it into a crank shape. He was right. Now it did turn . . . . His first attempts to induce the electrical current held similar disappointment, which he exploited with the same instinct" . . . . Or on Einstein "For ten years, Einstein thought on the matter. And like Descartes he found his insight in himself in a paradox which had first come to mind when he was sixteen. Suppose he were able to travel with a beam of light at its own velocity . . . ."

The views of the author, always original and provocative, may appear controversial to some—in particular, the main thesis, a denial of the view that an understanding of Nature can be reached by means of a mystical or subjectionist approach. In conclusion, this writer would like to say the reading of Gillispie's book has been for him a rewarding experience and an intellectual treat.

HENRI AMAR
The Franklin Institute Laboratories and Temple University

ENGINEERING MECHANICS: DYNAMICS, by Irving H. Shames. 631 pages, diagrams, 6 × 9 in. New York, Prentice-Hall, Inc., 1960. Price, $6.75.

A prominent engineer whose training was acquired during the time of the First World War recently commented to this reviewer on the increasingly scientific contents of engineering texts on dynamics. At that time, a discussion of Coriolis accelerations would have been given in the last few pages, while today it is found even in undergraduate texts among the kinematic preliminaries, where it belongs.

The present book is another example of this trend, and should be welcomed for its emphasis on engineering as Applied Physics. The author makes a highly successful attempt to present much of the theory of higher mechanics in a form useful to the engineer. The book is in the main a vector treatment of particle and rigid dynamics, which is introduced by a well-written long introductory chapter on kinematics. By devoting separate chapters to momentum and energy methods, the author emphasizes and distinguishes these two very useful and powerful approaches for solving dynamic problems.

The discussion of Euler's equations and their application to specific mechanical systems is particularly well presented, and the difficult subject of the spinning top is greatly clarified by the type of artistic diagrams contained in Goldstein's "Classical Mechanics,"

which aid the intuition in the visualization of complex three-dimensional motion.

A chapter on rectilinear motion is perhaps misnamed, since it is a condensed account of vibrations of single-and-multi-degree-of-freedom systems. The inclusion of a section on the motion of charged particles in the general discussion of particle motion seems justifiable in a book on Engineering Science. The final chapter on Momentum Equations for Deformable Media presents fluid dynamics as a logical continuation of the main subjects of the book, but one cannot help wondering whether a brief introduction to numerical methods, and at least a preview of more advanced topics such as generalized co-ordinates, and La Grange's formulation might not have formed a more stimulating conclusion.

But this is a minor criticism. The writing is pleasantly informal but clear, though the proof reading has been somewhat careless. Many of the problems, particularly those on central force motion, have topical interest, and a number of examples are solved by alternate methods in the text, in order to illustrate the importance of formulation. Answers to all problems are given.

If one were to take issue with the author on his otherwise excellent text, it would be on the score that he makes a complex subject appear almost too easy to understand, although he scrupulously points out the limitations of some approaches to the simpler cases. "Dynamics" should give the student a good introduction to an increasingly important area of Engineering Mechanics.

KLAUS L. CAPPEL
The Franklin Institute Laboratories

IONIZATION PHENOMENA IN GASES, by Gordon Francis. 300 pages, diagrams, 5½ × 8½ in. New York, Academic Press, 1960. Price, $10.50.

This is an old area of investigation regaining a new and increasing interest. It embraces or uses a large variety of disciplines ranging from spectroscopy and kinetic theory to magnetohydrodynamics and plasma oscillations. The author, an active investigator, has been working closely with Von Engel, who has been a pioneer in this field since the time of publication of his two-volume work "Elektrische Gasentlardungen" (Berlin, Springer, 1932, 1934).

The book begins with an elementary review of kinetic theory, spectroscopic notation and of "fundamental processes." This review may be considered too elementary by many in view of the more sophisticated material in the second half of the book. Particularly commendable are the chapters on the Similarity Principle (essentially the same material as that contained in the author's article in Vol. 22 of the new "Handbuch der Physik," 1956), on Alternating High Frequency Discharges and on the Upper Atmosphere. There is also a good introduction to the more modern topics, namely, Thermonuclear Effects and Plasma Oscillations.

A bibliography and a selected list of reference books are given at the end of each chapter, while six appendices provide some useful data for the reader. The book is well written and is probably the only book available on this topic (at this intermediate level) that is suitable as a textbook.

HENRI AMAR
The Franklin Institute Laboratories and Temple University

ELECTRICAL ENGINEERING SCIENCE, by Preston R. Clement and Walter C. Johnson. 588 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1960. Price, $9.50.

This text, intended for use in a two-semester introductory course for electrical engineering majors, is admirably suited to its purpose. Throughout, it gives every evidence of careful preparation, full consideration of the background and future needs of the student, and thorough "shaking down" by classroom use. The emphasis is on fundamental physical principles, developed by a judicious blending of the deductive and inductive methods.

This emphasis is as it should be, but yet apparatus and practice are not neglected. Indeed, the problem material gives special weight to these aspects. The result is a very nice balance.

The basic concepts on which the presentation is built are forces and fields, in both free space and material media, leading to Maxwell's equations in their integral form. Their differential form is never introduced, so that consideration of electromagnetic waves is necessarily omitted, but the omission cannot be considered serious at this level.

Material which is not commonly found in introductory texts includes theory of conduction, including semiconductors; network topology; frequency characteristics, transient response, and zero-pole configurations; and Fourier series and integrals. The introduction of these ideas early in the students' program should be welcome, since it will provide a foundation of familiarity for much of their later work.

A. C. BYERS
The Franklin Institute Laboratories

CRYOPHYSICS, by K. Mendelssohn. 183 pages, illustrations, 5 × 8 in. New York and London, Interscience Publishers, Inc., 1960. Price: $4.50 (cloth); $2.50 (paper).

This brief monograph is a comprehensive and up to date (1959) survey of low temperature physics, by one of the leading investigators in this field. No treatment in depth can be expected in such a short monograph. The author has succeeded, however, in writing an excellent introduction to this relatively new field, at an intermediate level.

After describing the general principles of cryogenics and thermometry, Mendelssohn treats the various phases of cryophysics in the following order: specific heats (pp. 34–50), magnetism (pp. 50–71), transport phenomena (pp. 71–92), superconductivity (pp. 92–126), the helium problem (pp. 127–160). The latter two subjects which constitute the core of the subject are more extensively covered. In the last chapter, the author deals with the "miscellaneous" such as trapped free radicals, and masses.

The style of exposition is clear and so are the descriptions of the experiments, although they are rather brief. The appendices, tables and illustrations provide a great help to the understanding of the text. The bibliography is brief, but carefully selected. Dr. Mendelssohn is to be commended for writing a skillful introduction which will stimulate students as well as scientists working in different fields.

HENRI AMAR
The Franklin Institute Laboratories and Temple University

THE EXPLORATION OF SPACE, edited by Robert Jastrow. 160 pages, illustrations, 7 × 10 in. New York, The Macmillan Co., 1960. Price, $5.50.

In the fast moving panorama of the space-flight sciences it is difficult to keep pace with the increasing cascade of rocket, satellite and probe results. Theories which in the past apparently have resolved or explained phenomena now fall short of providing definitive answers. It is for this reason that symposia are held to reveal the findings which may not only provide answers but shape the dramatic experiments and investigations of the future. One such symposium on Space Physics was held in Washington on April 29–30, 1959, sponsored by the National Academy of Sciences, the American Physical Society and the National Aeronautics and Space Administration.

The Exploration of Space represents the collected papers and discussions of this two-day meeting. Dr. Robert Jastrow in his introductory remarks defines the objectives of this meeting as:

"1. To awaken the interest of the scientific community in the problems of space research;

"2. To present an estimate of our present and future capabilities for space exploration, including the study of the upper atmosphere, the exploration of the moon and near-by planets, and the observation of the sun and stars from orbiting space platforms;

"3. To acquaint the experimentalist with existing instrumentation in space physics and to challenge his ingenuity in the construction of new apparatus."

To this end the ranking, articulate scientists in astronomy, geophysics and particle physics were invited to present their results, their present undertakings and their future aspirations. The stature of the participating scientists insures the authority and balance of the contributed papers.

To review a book where 13 authorities present a dynamic picture of their specific disciplines is a formidable task. The reviewer could run through the papers one by one to indicate the scope and significance of the paper. Were this to be done, and Dr. Jastrow has indeed done this in some 3000 words as a part of his introductory remarks, the editor of this JOURNAL would be most reluctant to use this review. For this reason, the reviewer will content himself with pointing up only a few of the provocative contributions.

Dr. Eugene Parker discusses the "solar wind" and its terrestrial effects. He proposes two experiments to pursue the investigations on the solar wind. In the first he would set up observing stations in space to detect and

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monitor the solar particles in transit, to study the structure of the shock waves at the fronts of the expanding gas clouds, and to measure magnetic field strengths. His second experiment would measure cosmic ray intensities at several distances from the earth to separate the geocentric and heliocentric modulating factors.

The imaginative N. C. Christofilos outlines the Argus Experiment and proposes other experiments one of which may resolve the question as to whether the inner radiation belt is fed by solar particles or by neutron decay products.

Dr. G. de Vaucouleurs offers a stimulating paper on Venus and Mars. He discloses that the nitrogen molecule has been verified in the Venusian atmosphere and also speaks about the configuration of the molecular bands at 3.5 microns due to the vibrations of the C-H bonds in organic molecules. Thus this should be final proof of the existence of plant life on Mars.

Dr. Leo Goldberg in a paper on Astronomy from Satellites and Space Vehicles outlines the astronomical researches which can be attempted in space. The problem in every case is outlined and the experiment for the solution is discussed.

Dr. Jastrow presents the closing paper on planetary atmospheres.

This book is a thin, relatively inexpensive book and yet in 154 pages it offers an almost complete survey of research in the space sciences. Between the covers of this book will be found a realistic blueprint of what tomorrow holds in store for the scientist. Both the scientist and layman must read this to stay abreast of the most dynamic influence on our lives.

I. M. LEVITT
The Fels Planetarium

PUBLICATIONS RECEIVED

ORGANIZING THE TECHNICAL CONFERENCE, by Herbert S. Kindler. 139 pages, 6 × 9 in. New York, Reinhold Publishing Corp., 1960. Price, $6.00.

WATER SUPPLY AND SEWERAGE, by Ernest W. Steel. Fourth edition, 655 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1960. Price, $11.00.

GEOLOGY: PRINCIPLES AND PROCESSES, by William H. Emmons, Ira S. Allison, Clinton R. Stauffer and George A. Thiel. 491 pages, illustrations, 8¼ × 9½ in. New York, McGraw-Hill Book Co., Inc., 1960. Price, $7.95.

HANDBOOK OF GEOPHYSICS, compiled by U. S. Air Force. Revised edition, v.p., illustrations, 8½ × 11 in. New York, The Macmillan Company, 1960. Price, $15.00.

CALIBRATION OF LINE STANDARDS OF LENGTH AND MEASURING TAPES AT THE NATIONAL BUREAU OF STANDARDS, by L. V. Judson. NBS Monograph 15, 12 pages, illustrations, 8 × 10½ in. Washington, U. S. Department of Commerce, 1960. Price, $0.15.

TRANSPORT AND ACCUMULATION IN BIOLOGICAL SYSTEMS, by E. J. Harris. Revised and enlarged second edition, 279 pages, diagrams, 5½ × 8½ in. New York, Academic Press, Inc., 1960. Price, $9.00.

MECHANICS AND PROPERTIES OF MATTER, by Reginald J. Stephenson. Second edition, 367 pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc., 1960. Price, $7.50.

ENGINEERING MATHEMATICS, by J. Blakey and M. Hutton. 603 pages, diagrams, 5¾ × 8½ in. New York, Philosophical Library, 1960. Price, $10.00.

TRANSFORMERS AND GENERATORS FOR POWER SYSTEMS, by R. Langlois-Berthelot. 541 pages, diagrams, 5½ × 8½ in. New York, Philosophical Library, 1960. Price, $12.00.

AN INTRODUCTION TO STOCHASTIC PROCESSES, by M. S. Bartlett. 312 pages, 5½ × 8½ in. New York, Cambridge University Press, 1960. Paper edition of 1955 first edition. Price, $2.95.

BOOK NOTES

CHEMICAL AND NATURAL CONTROL OF PESTS, by E. R. de Ong. 244 pages, illustrations, 6 × 9 in. New York, Reinhold Publishing Corp., 1960. Price, $7.50.

This is a sensible treatment of the broad problem of insect and plant-disease control, written by a man who knows his agriculture and his pesticides. The book strikes a workable balance between natural control and chemical control, and its twelve chapters contain sound advice not only for the people in the agricultural industry, but also for the home owner who finds himself bombarded with promotional material on scores of pesticides which may or may not benefit his lawn, trees, shrubs and flowers. This is a book which will be of use to manufacturers, farmers, home owners, sanitation engineers, and ranchers.

REACTOR HANDBOOK, VOLUME I, MATERIALS, edited by C. R. Tipton, Jr. 1207 pages, illustrations, 7¼ × 10 in. New York, Interscience Publishers, Inc., 1960. Price, $36.50.

This is the first volume of a four-volume revised and enlarged edition of the 1955 *Reactor Handbook*, prepared for the United States Atomic Energy Commission. Developments in nuclear technology and substantial declassification of nuclear data since 1955 have necessitated expansion of nearly all topics covered in the first edition. In addition, many new sections are included. The book is up-to-date through the spring of 1958, and extensive reference lists are included for most chapters—a feature which is new in the revised edition. Over 125 specialists are listed as contributors to this volume on *Materials*, and, although Tipton is listed as Editor on the flyleaf, six other men served as editors of the several parts comprising the book.

Fifty-three chapters are divided into seven Parts: Part A, General (5 chapters on health, safety, radiation damage, etc.); Part B, Fuel Materials (12 chapters covering such topics as solid fuels, uranium, plutonium and thorium and their alloys, ceramics, liquid-metal fuels, aqueous fuel systems, and molten salts); Part C, Cladding and Structural Materials (17 chapters on thirteen metals and their alloys, special-purpose alloys, carbides and cermets, and materials for use in valves and bearings); Part D, Control Materials (6 chapters dealing with engineering requirements for control materials including hafnium, boron, cadmium, rare earths, and silver); Part E, Moderator Materials (6 chapters covering light and heavy water, graphite, beryllium, organic materials, and hydrides and hydroxides); Part F, Coolant Materials (3 chapters, one of which is on gas coolants); and Part G, Shielding Materials (4 chapters). The 32-page appendix lists references for binary constitutional diagrams and should be of great value to reactor engineers.

Forthcoming volumes in the *Handbook* will cover Fuel Reprocessing, Engineering, and Physics and Shielding. The *Handbook* is well printed and well-bound. It will be of great aid to reactor engineers who are fortunate enough to have access to it through a library —its understandably high price will no doubt keep it off the shelves of an individual's library.

RAMIFICATION THEORETIC METHODS IN ALGEBRAIC GEOMETRY, by Shreeram Abhyankar. 96 pages, 7 × 10 in. Princeton, Princeton University Press, 1959. Price, $2.75 (paper).

The author has prepared this study from notes of a lecture course he gave at Columbia in 1955–56. The material is divided into five chapters: General Ramification Theory, Valuation Theory, Noertherian Local Rings, Two-Dimensional Local Domains, and Varieties and Transformations. The author has omitted proofs in many places, but he has done it advisedly, to provide what he calls a "short path" to the heart of the subject, which in many other treatments is never reached because of the inordinate amount of exposition. A simple proof of the local uniformization theorem, without the use of continued fractions, is a novel feature.

FIGURES OF EQUILIBRIUM OF CELESTIAL BODIES, by Zdeněk Kopal. 135 pages, 6 × 9¼ in. Madison, University of Wisconsin Press, 1960. Price, $3.00.

Kopal, Head of the Department of Astronomy, University of Manchester, herein extends Clairaut's 18th-century theory relative to the gravitational potential of a fluid mass subject to a given disturbing force. In his approach to the study of effects of gravitational interaction among celestial bodies, Kopal concentrates on the "construction of explicit expressions for the *potential* of our configuration," rather than using the more widely accepted perturbation methods of solving such problems. Kopal's work in extending Clairaut's theory to terms of second order was stimulated by the advent of artificial satellites which, he says, "has made it indispensable to refine our knowledge of the Earth's gravitational potential to at least the second-order terms." Chapters III and IV, on Second-Order Theory and Interaction Phenomena, respectively, are, for the most part, previously unpublished results of the author's investigations on equilibrium tides, rotational effects, effects of distortion on internal structure and the interaction between rotation and tides. These investigations grew out of his earlier work on distortions of stellar bodies.

FUNDAMENTALS OF ROCKET PROPULSION, by Raymond E. Weich, Jr. and Robert F. Strauss. 135 pages, diagrams, 6 × 9 in. New York, Reinhold Publishing Corp., 1960. Price, $5.50.

The first volume of a new series on Space Technology, this small book begins at the beginning of the subject—the mechanisms which make space flight possible. Eight chapters discuss various aspects of the rocket engine, such as reaction propulsion, nozzles for acceleration of the gases, liquid propellants, solid propellants, satellites and their orbits, and rockets of the future (nuclear, arc jet, plasma, ion, and photon). Two appendices provide the reader with a list of symbols and a set of rocket engine formulas. Subsequent volumes in the series will be more specific in scope than the present one, and will cover such subjects as rocket testing, space environment, space navigation and human factors.

MATERIALS SELECTION FOR PROCESS PLANTS, by Russel E. Gackenbach. 318 pages, illustrations, 6 × 9 in. New York, Reinhold Publishing Corp., 1960. Price, $8.50.

The author, a materials engineer for American Cyanamid, has assembled a useful handbook for those in charge of process plants, where increased equipment life, longer production runs and decreased maintenance costs are important. Corrosion-resistant and mechanical properties of several kinds of materials—metals, plastics, rubbers, paints and cements—are considered along with their fabricating characteristics. Each chapter covers one material thoroughly, including properties, sources and uses.

CYBERNETICS AND MANAGEMENT, by Stafford Beer. 214 pages, diagrams, 5½ × 8½ in. New York, John Wiley & Sons, Inc., 1959. Price, $4.50.

As the author states in his preface, this book is not a text-book nor a technical treatise, but a general exposition of the new science of cybernetics, aimed principally (but not exclusively) at management. Technical terms are carefully explained, and the non-scientific reader is gently led through the history, present uses and future possibilities of brain-like control machines. The author develops skilfully his thesis that in cybernetics may lie the salvation of the West.

LECTURES ON FOURIER INTEGRALS, by Salomon Bochner. 333 pages, 7 × 10 in. Princeton, Princeton University Press, 1959. Price, $5.00 (paper).

This translation of Bocher's classic book on Fourier integrals and of the supplement thereto has been published as Study 42 in the Annals of Mathematics Studies. As nearly thirty years have elapsed since the original German text appeared, the translation will be of particular interest to the present generation of mathematicians. Nine chapters form the main body of the book; they deal with basic properties of trigonometric integrals, the Fourier integral theorem, generalized trigonometric integrals, analytic and harmonic functions, functions of several variables, etc. The supplement covers monotonic functions, Stieltjes integrals and harmonic analysis.

PROPERTIES OF ELEMENTAL AND COMPOUND SEMICONDUCTORS, edited by Harry C. Gates. 340 pages, illustrations, 6 × 9 in. New York, Interscience Publishers Inc., 1960. Price, $8.50.

Volume 5 of the Metallurgical Society Conference series contains the 21 papers presented at a conference organized by the Semiconductors Committee of The Metallurgical Society. The purpose was to discuss those aspects of semiconductors technology which are closely related to metallurgy. The papers are divided into four parts: Metallurgical and Chemical Aspects (5 papers); Solid State and Surface Reactions (5 papers); Chemical and Lattice Defects (4 papers); and Physical Properties and Device Applications (6 papers). The book concludes with a panel discussion on The Role of Dislocations in Device Properties.

PRINCIPLES OF OPTICAL CRYSTALLOGRAPHY, by A. V. Shubnikov, translated from the Russian by Consultants Bureau Enterprises, Inc. 186 pages, diagrams, plates, 6 × 9 in. Price, $9.50.

The original Russian text of this book appeared in 1958. The author is Director of the Institute of Crystallography in the USSR, where intensive research is being done in this field. The book follows the course on optical crystallography given at Moscow University, covering isotropic transparent media, double refraction, interference of light in crystal plates, rotation of the plane of polarization, elliptical polarization and elliptical double refraction, and double absorption of light in crystals. Descriptions of apparatus and experimental methods are not included, since this is a theoretical text.

FLAT ROLLED PRODUCTS II: SEMI-FINISHED AND FINISHED, edited by E. W. Earhart and R. D. Hindson. 150 pages, illustrations, 6 × 9 in. New York, Interscience Publishers, 1960. Price, $4.00.

Volume 6 of the Metallurgical Society Conference series is the proceedings of the Second Annual Conference sponsored by the Mechanical Working Committee. This conference dealt with the process metallurgy of flat rolled steel products, with emphasis on semi-finished and finished products. Three papers deal with processing to semi-finished products (hot machine scarfing, selection of ingot and slab sizes, and scale structures) and four cover finished products (temper rolling, unitized automobiles, and processing and properties of magnetic materials). Discussions follow each paper. The editors and publishers are to be commended for having the printed proceedings available within six months of the conference date.

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NATIONAL BUREAU OF STANDARDS NEWS

UNUSUAL SOLAR DISTURBANCE

Members of the solar research group of the Radio Warning Services Section of the National Bureau of Standards' Boulder (Colo.) Laboratories have been appraising an unusual solar event which occurred on June 9, 1959. A severe radio blackout of very long duration, and large radio noise outbursts on a number of wavelengths, began at 1630 hours, UT. No solar flare could be seen on the sun's disk, although a prominent flare would normally be apparent at the time of the radio disturbance. It now appears that the reason for this anomaly can only be conjectured until solar behavior is known in much greater detail than at present.

The understanding of the association between solar events, ionospheric disturbances, and geomagnetic storms has played an important part in the Bureau's radio propagation prediction services, particularly in the radio disturbance warning network. Part of this prediction work, based on the analysis and evaluation of solar radio data received from many sources, is the determination of the relationships between various solar events, radio fadeout, and temporary changes in the earth's magnetic field. The unusual solar event of June 9, completely at variance with the experience of many years, has caused the Bureau to question the validity of some of what were considered to be established relationships.

As soon as the severity of the blackout of June 9 became apparent, radio and optical observations were intensified. The initial position of the radio-burst source on the sun's disk was established, by high resolution scan at 10.7 cm, to be N 24, E 90.[1] In the light of the H-alpha (α) spectral line, only jets and bright loops were observed at this position. It was not until almost 1½ hours after the first event that the expected Hα flare became observable at N 19, E 90. Ordinarily the flare is seen at the same time as the radio disturbance is observed.

Complete blackout of the Bureau's radio station, WWV, occurred at a number of receiving locations. Cosmic noise absorption of an outstanding nature (3 +) was evidenced by the great drop-off in received signal strength, and also by the unusually slow onset of the absorption. Solar radio bursts at 18 Mc were classified as 3 + in importance, and bursts at high frequencies (4.3 mm, 10.7 cm.) were very strong. These observations classify the ionospheric events as of relatively great importance (3 +).

Past observations of such outstanding events have, in almost all cases, revealed the presence of an Hα flare of major importance. The absence of such a flare in itself establishes the June 9 event as a most unusual one. The anomaly raises a number of serious problems concerning the understanding of solar-terrestrial relationships.

[1] "The 4.3 mm and 10.7 cm Outbursts of June 9, 1959," by J. R. Coates, A. E. Covington and S. Edelson, paper presented at the 103rd meeting of the American Astronomical Society, University of Toronto, 1959.

Both of the Hα jet ejections that took place appeared to be based on the back side of the sun. The jet phenomenon, together with unusually late appearance of the flare itself, led to the speculation that an early stage of the flare itself also occurred on the back side of the sun and in closer time-association with the earlier events mentioned. There is, of course, no way of establishing the "back-side" flare as a fact.

Assuming such a "back-side" flare presents still another problem: ascertaining how the solar radio emission, and the ionizing emission which produced the radio fadeout, could have reached the Earth if they were generated in the same solar region as the unobservable Hα flare. Normally, both emissions, Hα and ionizing radiation, travel in essentially straight lines. For these emissions to come from the same source, it would be necessary to assume a different trajectory for the back-side Hα emission than for the ionizing radiation, a conclusion at variance with many prior observations. However, it may be inferred that the Hα flare originated at a lower level in the sun's atmosphere than the source of the emissions. This inference would help explain why the Hα flare was not observed, although the ionizing radiation was able to reach the Earth.

Another unusual feature of the events of June 9 was the exceptionally low velocity of the radio emission source. The velocity was deduced from observations at two separate low frequencies. Slow drift bursts on dynamic spectrum records normally show velocities of the order of 1000 km per second at metric wavelengths.[2] By comparison, velocities derived from 18-, 38-, and 200-Mc fixed-frequency observations during the June 9 events average about 250 km per second.

An additional complication, a point of great interest which is under current study, is the lack of subsequent major geomagnetic and ionospheric storms. The Earth's magnetic field registered no major disturbance in the following few days.[3] Under ordinary circumstances, such disturbances would be expected.

This fact, coupled with the other apparent anomalies in the solar event of June 9, has forced a critical re-appraisal of past techniques of observation and earlier conclusions from data. The resulting inquiry can be expected to lead to increased understanding of the relationships between the individual components of associated solar events, and a consequent improvement in radio propagation prediction methods.

CALCULATION OF FALLOUT GAMMA RADIATION WITHIN STRUCTURES

A method for calculating fallout gamma radiation intensities within structures has been developed by C. Eisenhauer and L. V. Spencer of the National Bureau of Standards working under the sponsorship of the Office of Civil and Defense Mobilization. The method makes possible simple calculations of intensities within basements, and it can be used to find, at the center of a structure, the intensity of radiation entering through windows. With further

[2] "The Radio Spectrum of Solar Activity," by A. Maxwell, G. Swarup and A. R. Thompson, *Proc. I. R. E.*, Radio Astronomy Issue, Vol. 46(1), p. 142 (1958).

[3] There was a sudden commencement on June 11, followed by moderately disturbed three-hour K figures: 5, 6, and 5.

refinement, it should provide an improved theoretical approach to the analysis of shielding properties of structures.

In general, for a simple structure, fallout radiation particles will be distributed uniformly over the roof of the structure and on the ground surrounding it. The radiation that reaches a detector inside can then be separated into that part coming through the walls and that coming through the roof. The intensity, or dose rate, of gamma radiation within the structure will depend mainly upon barrier attenuation (a function of barrier thickness and density) and geometrical attenuation (a function of the solid angle subtended at the detector by the barrier).

The geometrical attenuation from sources on the roof can be described in terms of a single geometry factor. The dose rate from roof sources is then equal to the product of the barrier factor (barrier attenuation) and the geometry factor.

The geometrical attenuation of radiation from sources on the ground is somewhat more complicated because the direct radiation behaves differently from the radiation that has been scattered in the walls. The direct radiation reaches the detector only from below, while scattered radiation may reach it from either above or below. In the present method, the geometry factor for radiation from the walls is therefore separated into a direct and a scattered component. The geometry factor for direct radiation has long been understood. Although the exact value of the geometry factor for the scattered radiation has not yet been calculated, it is known to lie within certain limits and to vary slightly with wall thickness. A suitable value may therefore be estimated. It is also known that the radiation emerging from very thin walls is almost entirely direct radiation, while that emerging from very thick walls consists almost entirely of scattered radiation. For very thin walls, then, the dose rate is obtained as the barrier factor times the geometry factor for the direct radiation; while for very thick walls, the dose rate equals the barrier factor times the geometry factor for scattered radiation. For walls of intermediate thickness, the dose rate is obtained as the sum of the two products, each weighted according to the relative amounts of scattered and direct radiation.

Although several experiments have been performed to measure dose rates inside structures surrounded by artificially radioactive sources, the results of these experiments do not provide a proper test of calculations made by the Bureau's method because of complicating factors such as the presence of floors and inner partitions in the building. However, data suitable for comparison were obtained from a controlled experiment made at the Army Chemical Warfare Laboratories using cobalt-60 sources distributed around a concrete blockhouse. The blockhouse is 12 ft. long, 12 ft. wide, and 8 ft. high. Data were taken at ground level, 3 ft. above ground, and 6 ft. above ground at the center of the structure for wall thicknesses of 4 and 8 in. of concrete. Although the data are somewhat limited, they verify the trends predicted by the calculations.

CURRENT TOPICS

Visual Observation of Superconductivity Effects.—An important new technique for the study of superconductivity has been developed at the General Electric Research Laboratory in Schenectady. Superconductivity is a property of certain materials in which all resistance to passage of an electrical current disappears at temperatures near absolute zero (−459°F.).

The new method permits direct visual observation of transformations between the normal and superconductive states. Superconductive devices are expected soon to find important applications in electronic computers, missile and aircraft guidance and control systems, and other electronic units.

The experimental technique, developed at the GE Research Laboratory by Dr. Warren DeSorbo with the assistance of W. A. Healy, employs an ingenious combination of magnetic and optical effects to make visible the changes taking place in the "intermediate state," a condition in which portions of the sample are superconductive and portions normal. The intermediate state is of great scientific as well as practical interest. Its structure and the manner in which it changes when subjected to a magnetic field are strongly affected by impurities and metallurgical treatment of the specimen. The ability to observe this state directly will provide better understanding of these factors and speed the development of practical electronic devices employing superconductivity.

The technique, which was developed from an approach originally used at the Naval Research Laboratory, has already produced a wealth of information, especially on the motions of normal and superconducting "domains" which could not be observed before. It is also a quick and accurate method of measuring certain basic magnetic properties of superconductors. Thus the technique is proving valuable because it permits both observation of previously unexplored phenomena and more rapid and complete study of known phenomena.

Dr. DeSorbo's method is based on the fact that a sufficiently strong magnetic field can change a material from superconducting to normal, but so long as the material remains superconductive it is a perfect magnetic insulator. The specimen to be observed is usually prepared in the form of a flat disc or plate, although thin, vacuum deposited films have also been studied in this manner. On top of the sample is placed a plate of a special cerium phosphate glass about one hundredth of an inch thick. When magnetized, this glass has the property of rotating the plane of polarized light. Monochromatic polarized light is beamed on the glass, and the reflected light is viewed through a polarizing filter which can be rotated to any desired angle.

The material under study is customarily cooled in liquid helium to a temperature of about 1.5° above absolute zero. At this temperature it is superconductive, but it can be changed back and forth between superconductive and normal by varying a magnetic field. At certain magnetic field strengths, the specimen is in the inter-

mediate state, with portions of it normal and portions superconductive.

Since the superconductive areas shield the portions of the glass plate directly above them from the magnetic field, these parts of the glass are not magnetized. But portions of the glass directly above normal areas in the sample are not shielded and are thus magnetized. The parts of the glass that are magnetized rotate the plane of polarized light they reflect, while the parts that are shielded do not. By adjusting the polarizing filter through which the reflected light is observed, the magnetized portions of the glass can be made to appear dark and the unmagnetized portions light, or vice versa. Since this pattern of light and dark corresponds exactly to the pattern of superconducting and normal metal in the sample, changes in the state of the sample can be observed and photographed directly.

Re-Startable Rocket Engine.—Bell Aerospace Corp. recently overcame one of the major problems associated with space ventures—development of a rocket engine that can be turned off and on at will to control the position and direction of space craft.

Under contract to Lockheed's Missile and Space Division, Bell's Aerosystem Co. engineers have repeatedly started, shut-down, and re-started a liquid-fueled engine under simulated conditions approaching a total vacuum. Designated the Bell 8081 engine, it is an advanced version of the model used in the current Discoverer satellite vehicles. Tests conducted in one of the high altitude test cells at the Arnold Engineering Development Center, Tullahoma, Tenn., have shown it is possible to restart an engine far out in space.

Another type of restartable engine, the Aerojet General AJ10-104 "Able-Star," has also been tested at AEDC. It has been used successfully with the Transit IA, IIB, and Tiros satellite vehicles.

Simulated conditions ranged from sea level to very high altitudes and temperatures from 30° to 100°F. Additional tests were conducted at very low temperatures.

Prior to igniting the engine, the pressure and temperature inside the steel test cell were pumped down to simulate space conditions. The engine was then fired and after a period of running was shut down. Varying intervals of "orbital coasting" were simulated during which the low temperature and pressure conditions were maintained. The engine was then restarted.

In all the starts, covering the entire operational envelope of the engine as presently conceived, the engine ignited successfully.

Re-start capability would make it possible for a satellite in orbit to reuse its final stage engine for return to Earth or change its orbit and the engine re-started for controlled landings.

The need for re-start capability has come about through the multi-staging of missiles and the resultant penalty in increased weight, extremely complex controls and requirements for over-all reliability. A single re-startable engine would eliminate these requirements and permit greater payloads with considerable increased reliability.

Re-start of rocket engines is of particular interest to the space traveler who is interested in changing orbit without the initial weight penalty of multi-staging. Once free of gravity, the size and weight of the final stage is of little consequence and a much lighter over-all vehicle would result if the same upper stage could be used to accomplish all the desired maneuvers with long coast periods.

Portable Radar System.—A new portable radar system, designed to look far behind enemy lines and provide photographic plots of battle information, has been developed for Army Combat troops. The system will sweep enemy-held territory in a 25-mile semicircle and, by periodic photographic plots, help determine whether enemy attack is impending, or a withdrawal is in progress. It will also detect deployment and patterns of movement and provide widespread and accurate detection of enemy movement.

Based upon designs originated by Project Michigan, experimental models of the radar were built by the Strand Engineering Co. of Ann Arbor. Project Michigan is a continuing research and development program being carried out at The University of Michigan's Willow Run Laboratories to increase the Army's capability in combat surveillance and target acquisition.

The vantage point radar works on the Doppler principle, which detects slight movements toward or away from the radar because of the change in frequency of the radio waves reflected from the moving object.

The entire radar system, including antenna, weighs approximately 600 lb. The antenna is somewhat larger than those used by orthodox battlefield radars, allowing the use of a low-frequency scanning signal which has improved brush and foliage penetration. (The system is designated as the AN/TPD-2.)

In actual operation the radar system would be transported by helicopter to a point overlooking enemy terrain, where a three-man crew would set up the three-piece antenna, and assemble the equipment. In a relatively short time, the radar can begin plotting movement of enemy targets and the information relayed to headquarters.

Powerful Computer for Project Saturn.—A new computer, the most powerful to be used in America's space program, has been put to work in Project Saturn. The computer will be a key design and development tool in building the free world's biggest space vehicle, according to Dr. Wernher von Braun, director of the George C. Marshall Space Flight Center at Huntsville, Ala. The IBM 7090 data processing system will be used by space scientists at Huntsville to help meet the Saturn super-booster's critical timetable.

The Saturn vehicle is being built to send multi-ton payloads into Earth orbit; around the moon and back; and into deep space. In the course of development work, the new solid-state computer is expected to achieve the most accurate and detailed trajectory simulations ever plotted.

Saturn's 1,500,000-lb.-thrust booster will be "flown" thousands of times within the 7090 before being placed on a launching pad next year. With the new equipment, Marshall Space Flight Center personnel can see within a few minutes how their design modifications would affect an actual flight into space.

The Marshall Space Flight Center's two 7090 computers are the first of this type to be installed by International Business Machines Corporation for scientific or commercial data processing. Modified versions of the system are being incorporated into the Ballistic Missile Early Warning System.

The increased computing ability will help the Marshall Space Flight Center explore new areas of research opened by the clustering of eight powerful rocket engines in the Saturn booster. Measurements made as the

booster generates up to 30,000,000 hp. in static firing tests will be fed into the IBM 7090 to analyze vibration and heat transfer caused by interaction of the engines. Fuel slosh in the booster's multiple tanks also will be under study.

A major research effort utilizing the 7090 capabilities will examine the problems of multiplexing the Saturn's fuel supplies. If one or more engines should fail, the fuel would be redirected to other engines to enable the vehicle to continue its mission. Using the computer, Dr. von Braun's group will determine the best way to do this at various stages of flight under varying conditions of flame-out.

Dr. Helmut Hoelzer, director of the Computation Division, and Charles L. Bradshaw, deputy director, reported that because of Saturn's size and complexity, the amount of information obtained from each static firing test for computer analysis is eight to ten times greater than in any previous space program. They said evaluation of telemetering data for a live flight of the single engine Redstone took as long as five to six weeks. It is expected the center's new 7090s will do the same job for the more complex Saturn in three or four days.

The 7090 is able to reduce the millions of pieces of information gathered from test firings into meaningful form quickly because of its operating speed. It is able to call information from its magnetic core storage unit in 2.18 millionths of a second. With the use of a new data multiplexor, the system simultaneously is able to read information from magnetic tape, write information on tape and compute.

To take full advantage of the computer's operating speed, the Marshall Space Flight Center Computation Division has devised an automatic system which enables the 7090 to monitor its own operations. It is called SPOOK—Supervisory Program Over Other Kinds—and represents an unseen hand telling the computer what to do at the right time to do it.

SPOOK consists of one reel of magnetic tape containing 50,000 instructions. As different types of work are processed, the computer will tell its operator what is being done and when tape reels must be changed.

There will be relatively few live tests of Saturn because of the vehicle's cost. Dr. Hoelzer, who has worked with Dr. von Braun in the field of rocketry and space flight for more than 20 years, said: "The V-2 rocket was developed at Peenemunde basically without automatic digital computers. As a result, there were approximately 1,000 test firings. Yet with the vastly more intricate Saturn, we have scheduled only 10 research and development firings.

"We now can simulate a trajectory in a few minutes on the 7090 for several hundred dollars. It would cost millions of dollars to stage a live flight. Yet by repeated flight simulation we can find out much of the information we would obtain from a launching."

To simulate Saturn flights, mathematical equations representing factors such as weather conditions and the rotation and gravitation of the earth and moon are stored within the 7090. Information about the design of the booster then is fed into the computer so that it can determine the trajectory the vehicle would take.

In addition to providing development assistance, the new systems also will have a direct effect on Project Saturn's computation costs. Dr. Hoelzer said the two 7090 systems will accomplish in eight hours what three large-scale vacuum tube machines accomplished in twenty hours. He said they will do this with a 25 per cent saving in machine time costs and a corresponding decrease in personnel time.

Liquid Propellant Gas Cartridge.—A liquid propellant gas cartridge no bigger than a flashlight battery but powerful enough to nudge a faulty jetliner landing gear has been developed by Olin Mathieson Chemical Corporation. It produces high pneumatic pressures in a fraction of a second, and can be adapted to operate nearly any pneumatic power-actuated device.

The liquid is a monopropellant containing its own oxydizer. Use of a liquid propellant as a source of power gives the cartridge certain advantages over its solid propellant counterpart:

1. *Clean burning characteristics.* The power system is left virtually free of carbon deposits. Gas produced by the cartridge will not cause corrosion of metals nor will it damage other materials used in a power system.

2. *Low flame temperature.* The liquid produces gas temperatures 50 per cent below those resulting from the burning of solids.

3. *Burns under varied conditions.* Reliable ignition conditions exist in the liquid propellant cartridge at temperatures between −109° and +212°F.

4. *High-ratio gas expansion.* Gas expands from liquid at a ratio as high as 14,000 to 1, producing extremely high pressures for pneumatic applications.

Any type of igniter, from the simplest percussion type primer to an elaborate electronic device, will work in the new cartridge.

"Cocktail Party" Effect Reproduced Electronically.—The psychoacoustic phenomenon called the "Cocktail Party Effect" has recently been duplicated electronically by a group of investigators at Bell Telephone Laboratories. This term has been applied to the ability of the human listener to concentrate on a specific voice in which he is interested, and thereby separate it from the surrounding babble of voices or other noise, even though it may be no louder than the surrounding noise level.

The binaural, or two-eared, listener is able to enhance the effective intensity of the desired voice by 5 to 15 decibels by suppressing the background noise level. A scheme for reproducing this feat electronically was recently described by E. E. David, Jr. and J. F. Kaiser of Bell Laboratories to the Acoustical Society of America, meeting in Providence, R. I.

In the cocktail party effect, the disparate sound-waves picked up by the two ears are used. The single ear, with only one sample of the sound field, cannot pick out specific voices or sounds from a surrounding babble as easily. A telephone transmitter, or any single microphone, is similarly unable to suppress spurious noises. Thus, background noise interferes much more with the voice of a person talking over the telephone or loudspeaker system than the same noise would if the two-eared listener were standing in the same room as the talker.

The gain achieved by the ear and brain in the cocktail party effect is greater than would be expected through simple linear addition of the two aural inputs. Therefore, nonlinear techniques were used in the attempts to duplicate the effect electronically. In one technique, the outputs of two microphones were compared, or cross-correlated, to generate a "gating wave" which was then applied to the combined output of the two microphones. The effect of this gating was to raise the intensity of the combined signal only when energy from the desired voice arrived simultaneously at both microphones. Otherwise, the gate suppressed the combined signal. Noise or interfering

speech signals passed through the gate if they occurred simultaneously with the desired speech signal. However, these events were rare and the pattern, or continuity, of the undesired signal was disrupted. Thus its interfering qualities were greatly reduced.

In subjective tests with such a processor using preliminary circuit concepts, enhancements of 9 db in effective signal levels were achieved against an interfering single-talker background. A smaller effect, 5 db, was found when two extra talkers interfered.

While the data reported by Messrs. David and Kaiser are the results of early experiments, they indicate that machine reproduction of some elementary properties of human hearing may not be as far in the future as previously expected.

Electron Behavior in Alloys.—The distribution of electrons in solid alloys consisting of one metal dissolved in another—a mystery whose unraveling might lead eventually to production of stronger metals—has puzzled scientists for years.

A recent report by Dr. T. J. Rowland, a scientist at the Metals Research Laboratories of Union Carbide Metals Company, division of Union Carbide Corporation, recently took an important step toward solving this mystery by studying how closely the electrons that accompany solute atoms surround these atoms in solid solution. By observing the effect of the alloying on the nuclear resonance of copper, Dr. Rowland has been able to determine how these extra electrons are grouped around the solute atoms.

Nuclear resonance is the result of the absorption of energy by nuclei from a radio frequency electromagnetic field. The amount of energy absorbed at any particular frequency depends in turn upon the electron distribution. The resonance amplitude of copper undergoes a sharp reduction upon alloying and is of special interest; its dependence upon solute valence and size argues decisively in favor of conduction electron charge redistribution as the dominant source of the electric field gradients surrounding these solutes.

Dr. Rowland's results combined with new theory, developed by W. Kohn and S. H. Vosko of Carnegie Institute of Technology, have shown actual distribution of the electrons is as a diffuse cloud around the solute atoms. The charge density in the cloud decreases about in proportion to the reciprocal cube of distance from the solute atom rather than exponentially and thus its effects are of longer range than had been supposed. Earlier workers have often confused these electronic effects with "local strains" that also happen to decrease as the reciprocal cube of distance from solute atoms.

This work is part of a concerted study of various aspects of the fundamental structure of metallic solid solutions in progress in UCM's laboratories under the joint sponsorship of Union Carbide Corporation and the Atomic Energy Commission. The objective of the study is to understand the interactions on an atomic scale of the different atoms that make up a metallic solid solution (alloy).

A clearer understanding of these interactions should lead eventually to methods of producing stronger metallic structures. Dr. Rowland's report can be considered as an important bit of information leading toward the understanding of the structure of alloys. Understanding the structure on an atomic scale appears to be essential for the design of superior materials on any but an empirical basis.

146-Decibel Acoustic Noise Facility Developed.—An acoustic noise facility capable of producing a higher sound pressure level than has ever been achieved in a large capacity reverberant chamber has been accepted by Sud Aviation, Cannes, France, from the manufacturer, Avco Corporation. Noise generators designed by Avco's Research and Advanced Development division of Wilmington, Mass., feed 146 decibels into a 200-cu. ft. pentagonal reverberant chamber where individual octave band sound pressure levels can be maintained within ±3 db with a specimen as large as 20 cu ft.

The major components of the acoustic test facility developed for Sud Aviation are a pentagonal reverberant chamber, noise generators, amplifiers, control console, and instrumentation.

Altogether, 36 noise generators and 21 power amplifiers are used in this system. Thirty-four of the noise generators are horn-type covering a 150 to 9,600 cps frequency range; the remaining ten are direct radiating cone niose generators in the 37 to 150 cps frequency range.

A pentagonal chamber was chosen by Avco because this shape produces uniform sound pressure levels. Experimentation with sound chambers designed for in-house work had shown Avco engineers that an essentially flat octave spectrum can be achieved when the test specimen is limited to one-tenth of the total chamber volume. The Sud Chamber will occupy a floor area approximately 7 ft. by 7 ft. and will be made of poured concrete.

The Avco acoustic noise generators have produced sound pressure levels as high as 170 db at discrete frequencies. The noise generator is essentially an electro-mechanical transducer of the moving coil type.

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GUIDE FOR THE PREPARATION AND PUBLICATION OF SYNOPSES *

1. "Synopsis" is a term adopted by the Royal Society of London (in fulfillment of a recommendation of the Scientific Information Conference sponsored by the Society in 1948) and by the Unesco International Conference on Science Abstracting, 1949, to describe an author's summary of a scientific paper which is published simultaneously with the paper itself after editorial scrutiny by the editor of the journal in which it is published.
2. The purpose of a synopsis is not only to convenience the readers of the journal in which it is published, but also to reduce the cost and to expedite the work of the abstracting journals, and thus to contribute to the general improvement of informational services in the scientific field.
3. The synopsis should comprise a brief and factual summary of the contents and conclusions of the paper, a pointer to any new information which it may contain, and an indication of its relevance. It should enable the busy reader to decide more surely than he can from the mere title of the paper whether it merits his reading it.
4. The author of every paper is consequently requested to provide also a synopsis of it, in accordance with the following suggestions.

STYLE OF WRITING

5. Use complete sentences rather than a mere list of headings. Any reference to the author of the article should be in the third person. Standard rather than proprietary terms should be used. Unnecessary contractions should be avoided. It should be presumed that the reader has some knowledge of the subject but has not read the paper. The synopsis should therefore be intelligible in itself without reference to the paper. (For example, it should not cite sections or illustrations by their numerical references in the text.)

CONTENT

6. As the title of the paper is usually read as part of the synopsis, the opening sentence should be framed accordingly so as to avoid repetition of the title. If, however, the title is not sufficiently indicative, the opening sentence should indicate the subjects covered. Usually, the beginning of a synopsis should state the objects of the investigation.
7. It is sometimes valuable to indicate the treatment of the subject by words such as: brief, exhaustive, theoretical, etc.
8. The synopsis should indicate newly observed facts, conclusions of an experiment or argument, and, if possible, the essential parts of any new theory, treatment, apparatus, technique, etc.

* The JOURNAL is publishing this Guide at the suggestion of the Abstracting Board of the International Council of Scientific Unions, in an effort to help Abstracting Journals in their monumental task. By following the procedures outlined here, the distribution of accurate and relevant scientific information can be vastly improved.

9. It should contain the names of any new compound, mineral species, etc., and any new numerical data, such as physical constants; if this is not possible, it should draw attention to them. It is important to refer to new items and observations, even though some may be incidental to the main purpose of the paper; such information may otherwise be hidden although in fact it might be very useful.
10. When giving experimental results the synopsis should indicate the methods used; for new methods the basic principle, range of operation and degree of accuracy should be given.

REFERENCES, CITATIONS

11. If it is necessary to refer in the synopsis to earlier work, the reference should always be given in the same form as in the paper; otherwise, references should be omitted.
12. Citations to scientific journals should be made in conformity with the standard practice of the journal for which the paper is written. (The International Conference on Science Abstracting has recommended the standard proposed by the International Organization for Standardization, Technical Committee 46, names of journals being abbreviated as in the *World List of Scientific Periodicals*.)

LENGTH

13. The synopsis should be as concise as possible. It should only in exceptional cases exceed 200 words, so as—among other things—to permit it, when printed, to be cut out and mounted on a 3 × 5 inch card.

PUBLICATION—LANGUAGE AND FORMAT

14. The International Conference on Science Abstracting has recommended that synopses be published in one of the more widely used languages, no matter what the original language of the paper, in order to facilitate its international usefulness.
15. The International Conference on Science Abstracting also commended the practice of certain journals in which all the synopses appearing in a single issue are printed together either inside the cover or with advertisements on the back in such a way that they can be cut out and mounted on index cards for reference without mutilating the pages of the journal itself. For this purpose the synopses should be not more than about 4 inches wide so as to be mounted on 3 × 5 inch cards.